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**AN APPLICATION OF THE FINITE ELEMENT
METHOD TO ELASTIC-PLASTIC PROBLEMS
OF PLANE STRESS**

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M. SALMON

*IIT Research Institute
Chicago, Illinois*

L. BERKE and R. SANDHU

*Air Force Flight Dynamics Laboratory
Wright-Patterson AFB, Ohio*

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FOREWORD

This report was prepared by the Illinois Institute of Technology Research Institute and the Air Force Flight Dynamics Laboratory as a joint in-house effort under Project No. 1467, "Structural Analysis Methods."

The computer program presented here was developed through a number of modifications during the period January 1967 through December 1968.

This report has been reviewed and is approved.



FRANCIS J. JANIK, JR.
Chief, Solid Mechanics Branch
Structures Division
Air Force Flight Dynamics Laboratory

ABSTRACT

A computer program is presented for the small strain analysis of plane structures in the strain hardening elastic-plastic range. The finite element displacement method is used to perform the linear analyses in the iterative scheme. Bar and constant strain isotropic plane stress triangles are available for use in constructing idealizations. The use of ten different sets of material properties, three different material laws, and incremental proportional loading are available as options. Good correlation is shown with available test data and theoretical solutions.

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SECTION I

INTRODUCTION

This report describes a computer program for the stress analysis of plane structures in the elastic-plastic range by the finite element method. The program can handle bar and triangular plate elements so that it is applicable to trusses and to the analysis of in-plane stresses in reinforced plates. The material behavior is assumed to be isotropic and the user has a choice of three types of stress-strain laws and ten different materials.

A numerical step by step procedure for obtaining solutions which satisfy the requirements of the incremental theory of plasticity for materials which obey the Mises yield condition and the associated flow rule is used in the program. At each step in the solution, an iterative procedure is used to find the correct values of the strain increments. Changes in plastic strain are accounted for by the addition of fictitious plastic forces to the actual loading on the structure in such a way that the deflections of the structure under the modified loading with assumed elastic behavior are equal to the actual deflections. A modified form of the computer program given in Reference 1 is used to obtain elastic solutions.

The finite element method originally developed for elastic stress analysis was extended to apply to inelastic material behavior by Padlog (Reference 2) et al. Further studies of the use of the method in elastic-plastic problems have been made, for example, at MIT (Reference 3) and the California Institute of Technology (Reference 4).

The method has been extended to apply to anisotropic materials by Jensen (Reference 5). Marcal and King (Reference 7) have applied the method to problems of plane stress and strain and to axisymmetric problems. However, the computer programs necessary for the application of the method have not been published. The purpose of this report is to make such a program available.

$\sigma_{ij} = \frac{1}{2} \epsilon_{ij}^T k \cdot \epsilon$, ϵ_{ij} , and k are unknown. Throughout this section,

The following section of the report presents a detailed explanation of the method of analysis. The application of the program is demonstrated by examples in Section III. Directions for the use of the computer program are given in Section IV. Listing of the program and sample data sets are given in the Appendix.

$$\begin{aligned} & \left[A_1^{-1} + A_1^{-1} A_2 A_2^{-1} I - A_1 A_1^{-1} (A_2^{-1})^T A_2 (I - A_1 A_1^{-1}) \right] \\ & \quad \times \left[(A_2^{-1} (I - A_1 A_1^{-1})^T A_2^{-1} (I - A_1 A_1^{-1}))^{-1} \right] \end{aligned}$$

It can be easily checked that a conditional inverse of $(A_1 A_1^{-1})$ is $(A_1 A_1^{-1})^T A_1 (A_1^{-1})^T$ and further that $(A A^{-1})^{-1} = (A^{-1})^T A^{-1} = A^{-1}$, $A^{-1} A A^{-1} = A^{-1}$, $A^{-1} A = A$. Thus

$$A_1^{-1} [e^T (A_1 A_1^{-1})^T A_1^{-1} - I] e^T = e^T (A_2^{-1} (I - A_1 A_1^{-1})^T A_2^{-1} (I - A_1 A_1^{-1}))^{-1} e^T$$

This completes the proof.

Lemma 5.2. Given $g \in W_p$, there exists a vector c^T in the row space of A_2 such that $c^T g$ is the B.I.U. of $c^T \xi_2$.

Proof: Let $\xi = (I - A_1 A_1^{-1})^T A_2 \perp \xi_2$, then $B(c^T \xi) = c^T A_2^{-1} (I - A_1 A_1^{-1})^T A_2 \xi = c^T \xi_2$ (say). Using the same argument as in the proof of the last lemma, the B.I.U. of $c^T \xi_2$ is

$$(5.5) \quad c^T \hat{\xi}_2 = I^T A_2^{-1} (I - A_1 A_1^{-1})^T A_2 [A_2^{-1} (I - A_1 A_1^{-1})^T A_2^{-1} (I - A_1 A_1^{-1})^T]^{-1} \xi_2.$$

SECTION II

METHOD OF ANALYSIS

The analysis procedure used in the computer program is described here. The method, first used by Padlog for the solution of problems involving plastic flow and creep, is given here in a slightly modified form. The well known formulas for the stiffness of bars and triangular plate elements are first presented. Then the step by step iterative procedure used for the solution of problems in which plastic flow occurs is described.

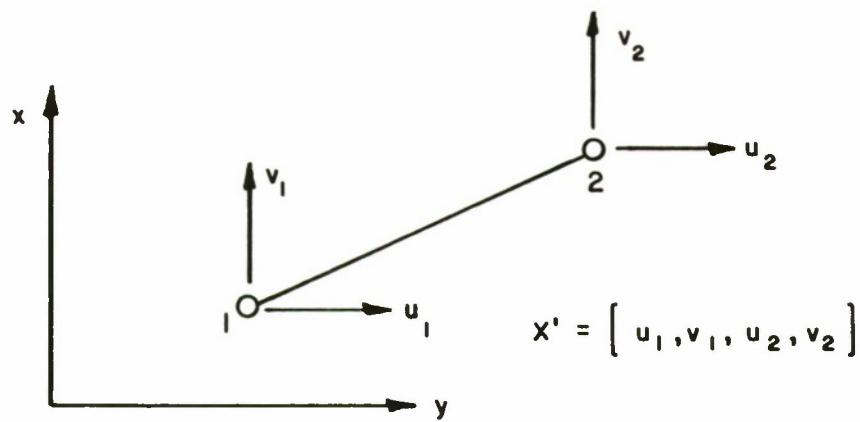
1. ELEMENT PROPERTIES

The two types of elements to be considered in this analysis (the bar and the triangular plate) are shown in Figure 1. The coordinates of the end points of the bar and the vertices of the triangle are referred to a fixed coordinate system in the plane. The cartesian components of the nodal displacements for each of these elements comprise the element displacement vector X . The ordering of the displacement components is shown in Figure 1. The total element strains designated by the vector ϵ can be expressed in terms of the nodal displacements by an equation of the form

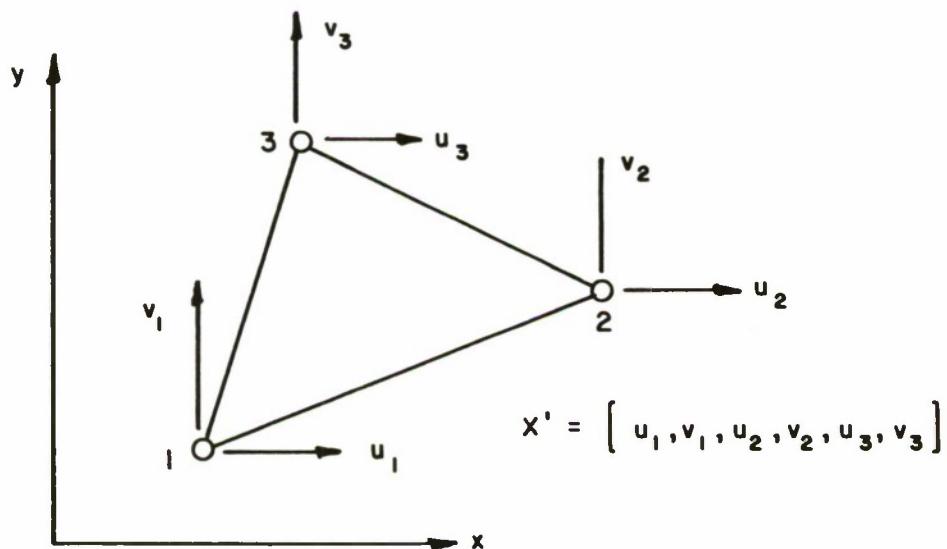
$$\epsilon = BX \quad (1)$$

The stresses are related to the elastic strains $\epsilon^e = \epsilon - \epsilon^p$ by Hooke's law

$$\sigma = C \epsilon^e \quad (2)$$



Bar Element



Triangular Plate Element

Figure 1. Bar and Plate Elements

The nodal forces \mathbf{F} corresponding to given displacements \mathbf{X} are found by the principle of virtual work. That is

$$\bar{\mathbf{X}}' \mathbf{F} = \int_V \bar{\epsilon}' \sigma dV \quad (3)$$

where $\bar{\mathbf{X}}$ and $\bar{\epsilon}$ are the virtual displacement vector and the corresponding strain vector, respectively, and the integration is carried out over the volume of the element. Prime superscripts denote transposed matrices. Using Equations 1 and 2 in Equation 3 gives the results

$$\bar{\mathbf{X}}' \left[\mathbf{F} - \int_V \mathbf{B}' \mathbf{C} \mathbf{B} dV \mathbf{X} + \int_V \mathbf{B}' \mathbf{C} dV \mathbf{\epsilon}^P \right] = 0 \quad (4)$$

Or, since the elements of $\bar{\mathbf{X}}$ are arbitrary, then

$$\mathbf{F} + \mathbf{F}^P = \mathbf{k} \mathbf{X} \quad (5)$$

where

$$\mathbf{k} = \int_V \mathbf{B}' \mathbf{C} \mathbf{B} dV \quad (6)$$

is the element stiffness matrix and

$$\mathbf{F}^P = \mathbf{D} \mathbf{\epsilon}^P \quad (7)$$

is the vector of plastic forces in which

$$\mathbf{D} = \int_V \mathbf{B}' \mathbf{C} dV \quad (8)$$

The definitions of the nodal force and displacement vectors and of the matrices \mathbf{C} , \mathbf{B} , \mathbf{D} , and \mathbf{k} for bars and triangular plate elements are:

Bar Element

$$\mathbf{X}' = [u_1, v_1, u_2, v_2] \quad (9)$$

$$\mathbf{F}' = [F_{x1}, F_{y1}, F_{x2}, F_{y2}] \quad (10)$$

$$\mathbf{B} = \frac{1}{L^2} [-x_{21}, -y_{21}, x_{21}, y_{21}] \quad (11)$$

in which L is the length of the bar and

$$\frac{x_{21}}{L} = \frac{x_2 - x_1}{L}, \quad \frac{y_{21}}{L} = \frac{y_2 - y_1}{L} \text{ etc} \quad (12)$$

$$C = E \text{ -- Young's modulus} \quad (13)$$

$$\text{Hence equation (5.8c) becomes} \quad D' = \frac{AE}{L} \begin{bmatrix} -x_{21}, -y_{21}, x_{21}, y_{21} \end{bmatrix} \quad (14)$$

where A is the cross-sectional area of the bar.

Now $\frac{AE}{L} = C$ by the definition of C . Thus since $D' = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$ we have
 $B_{11} = C$ and equation $k = \frac{AE}{L^3} \begin{bmatrix} k_{11} & -k_{11} \\ -k_{11} & k_{11} \end{bmatrix}$ is equivalent to
 $D' = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$

in which

that (5.12a) and (5.12b) indicate that the equation $D' = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$ is always associated with the element rows of B . The rows of B consist of parts of the columns of D' .

This can be found that in theorem 5.1, the condition (5.8b) may be omitted.

Triangular Plate Element

From theorem 5.1 we have established

$$x' = [u_1, v_1, u_2, v_2, u_3, v_3] \quad (17)$$

Designate in (17) with respect to (5.8a) as the equation (5.8a) be replaced by

$$F' = [F_{x1}, F_{y1}, F_{x2}, F_{y2}, F_{x3}, F_{y3}] \quad (18)$$

where x_i, y_i ($i = 1, 2, 3$) are the coordinates of the vertices of the triangle and $\epsilon' = [\epsilon_x, \epsilon_y, \gamma_{xy}]$ has three columns. Write

$$\sigma' = [\sigma_x, \sigma_y, \tau_{xy}] \quad (20)$$

$$B = \begin{bmatrix} -y_{32} & 0 & y_{31} & 0 & -y_{21} & 0 \\ 0 & x_{32} & 0 & -x_{31} & 0 & x_{21} \\ -x_{32} & -y_{32} & -x_{31} & y_{31} & x_{21} & -y_{21} \end{bmatrix} \quad (21)$$

The left hand side of (5.8a) equals

$$h = x_{21}y_{31} - x_{31}y_{21} \quad (22)$$

Hence theorem (5.8a) equation (22) is equivalent to

The absolute value of h equals twice the area of the triangular element.

$$C = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \quad (23)$$

in which ν = Poisson's ratio

$$D = \frac{E t |h|}{2h(1-\nu^2)} \begin{bmatrix} -y_{32} & -\nu y_{32} & C_1 x_{32} \\ \nu x_{32} & x_{32} & -C_1 y_{32} \\ y_{31} & \nu y_{31} & -C_1 x_{31} \\ \nu x_{31} & -x_{31} & C_1 y_{31} \\ -y_{21} & -\nu y_{21} & C_1 x_{21} \\ \nu x_{21} & x_{21} & -C_1 y_{21} \end{bmatrix} \quad (24)$$

where $C_1 = (1-\nu)/2$ and t is the element thickness.

$$k = \frac{E t}{2|h|(1-\nu^2)} \bar{k} \quad (25)$$

where

$$\bar{k}_{ij} = \bar{k}_{ji} \quad (26)$$

$$\begin{aligned}
 \bar{k}_{11} &= y_{32}^2, \quad \bar{k}_{12} = -\nu x_{32} y_{32} - C_1 x_{32} y_{32} \\
 \bar{k}_{13} &= -y_{31} y_{32} - C_1 x_{31} x_{32}, \quad \bar{k}_{14} = \nu x_{31} y_{32} + C_1 y_{31} x_{32} \\
 \bar{k}_{15} &= y_{21} y_{32} + C_1 x_{21} x_{32}, \quad \bar{k}_{16} = -\nu x_{21} y_{32} - C_1 y_{21} x_{32} \\
 \bar{k}_{22} &= x_{32}^2 + C_1 y_{32}^2, \quad \bar{k}_{23} = \nu y_{31} x_{32} + C_1 x_{31} y_{32} \\
 \bar{k}_{24} &= -x_{31} x_{32} - C_1 y_{31} y_{32}, \quad \bar{k}_{25} = -\nu y_{21} x_{32} - C_1 x_{21} y_{32} \\
 \bar{k}_{26} &= x_{21} x_{32} + C_1 y_{21} y_{32}, \quad \bar{k}_{33} = y_{31}^2 + C_1 x_{31}^2, \\
 \bar{k}_{34} &= -\nu x_{31} y_{31} - C_1 y_{31} x_{31}, \quad \bar{k}_{35} = -y_{21} x_{31} - C_1 x_{21} x_{31} \\
 \bar{k}_{36} &= \nu x_{21} y_{31} + C_1 y_{21} x_{31}, \quad \bar{k}_{44} = x_{31}^2 + C_1 y_{31}^2, \\
 \bar{k}_{45} &= \nu y_{21} x_{31} + C_1 x_{21} y_{31}, \quad \bar{k}_{46} = -x_{21} x_{31} - C_1 y_{21} y_{31} \\
 \bar{k}_{55} &= y_{21}^2 + C_1 x_{21}^2, \quad \bar{k}_{56} = -\nu x_{21} y_{21} - C_1 x_{21} y_{21} \\
 \bar{k}_{66} &= x_{21}^2 + C_1 y_{21}^2
 \end{aligned} \tag{27}$$

2. ELASTIC-PLASTIC ANALYSIS

In the elastic range of material behavior the equilibrium equations for a structure composed of plate and bar elements of the type considered here can be written

$$\mathbf{F} = \mathbf{KX} \tag{28}$$

where the force and displacement vectors now have as their components the cartesian components of force and displacement at all the nodes and \mathbf{K} is the assembled stiffness matrix for the whole structure. The solution of Equation 28 for the unknown displacement is given symbolically by

$$\mathbf{X} = \mathbf{K}^{-1} \mathbf{F} \tag{29}$$

The displacements known, the element strains can be obtained from Equation 1 and the stresses from Equation 2. However, when the stresses reach the intensity required to cause plastic flow, it becomes necessary to determine the increments of plastic strain caused by the load increment. The material is assumed to obey the Mises yield condition and the associated flow rule. For plane stress the following equations apply

$$\bar{\sigma} = (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3 \tau_{xy}^2)^{1/2} = H(\bar{\epsilon}^p) \quad (30)$$

$$\Delta \bar{\epsilon}_x^p = \frac{2}{\sqrt{3}} (\Delta \epsilon_x^p)^2 + \Delta \epsilon_x^p \Delta \epsilon_y^p + \Delta \epsilon_y^p + \frac{1}{4} \gamma_{xy}^p)^{1/2} \quad (31)$$

$$\left. \begin{aligned} \Delta \epsilon_x^p &= \frac{\Delta \bar{\epsilon}^p}{2 \bar{\sigma}} (2 \sigma_x - \sigma_y) \\ \Delta \epsilon_y^p &= \frac{\Delta \bar{\epsilon}^p}{2 \bar{\sigma}} (2 \sigma_y - \sigma_x) \\ \Delta \gamma_{xy}^p &= 3 \frac{\Delta \bar{\epsilon}^p}{\bar{\sigma}} \tau_{xy} \end{aligned} \right\} \quad (32)$$

where $\bar{\sigma}$ and $\bar{\epsilon}^p$ are the effective stress and the effective plastic strain, respectively, and where $H(\bar{\epsilon}^p)$ is the stress-plastic strain relation for uniaxial stress.

If it is assumed that the response of the structure to the removal of a load increment will be completely elastic then Equation 28 can be modified to account for plastic flow as follows

$$K X = F + F^p \quad (33)$$

where X and F are the displacement and load after the application of the increment and F^p is the vector of plastic forces corresponding to

the plastic strains. The plastic strain increments caused by the increment of load must satisfy Equations 32 and for an element undergoing plastic flow the stresses must satisfy the yield condition (Equation 30).

The following step by step iterative method is used to obtain solutions:

1. An increment is given to the applied loads.
2. New values of displacement are found from Equation 33 using the current values of the plastic forces (these will be zero for the first step).
3. The displacements are used to compute total strains, elastic strains, stresses, and the effective stress.
4. If the new value of the effective stress is greater than the largest previous value, the element is plastic and the effective stress is used to determine a new value of the effective strain.
5. Plastic strain increments computed from Equations 32 are added to the current values of the plastic strain and new values of the plastic forces are calculated.
6. If the increment in effective plastic strain is sufficiently small the iteration is complete and a return to step 1 is made, if not a return is made to step 2 and a new cycle begun.

This procedure is applied to each of the elements and the decision to start a new step (apply a load increment) is based on the largest plastic strain increment found among all the elements.

An important feature of the method is the way in which the effective plastic strain is computed from the new value of the effective stress at each iteration. If the inverse of Equation 30 is used to give $\bar{\epsilon}^p$ as a function of $\bar{\sigma}$ the solution may become unstable. This becomes obvious when one considers the case of the elastic, perfectly plastic material for which the inverse of the function $H(\bar{\epsilon}^p)$ does not exist. To avoid this difficulty the "constant strain" method of Reference 2 is used. In this method the total strain ϵ_t is taken equal to the sum of the value of $\bar{\epsilon}^p$ computed in the previous iteration and $\bar{\sigma}/E$.

The stress-strain law can be written in the form

$$\epsilon_t = \frac{\bar{\sigma}}{E} + \bar{\epsilon}^p$$

or

$$\epsilon_t = \frac{H(\bar{\epsilon}^p)}{E} + \bar{\epsilon}^p \quad (34)$$

The new value of $\bar{\epsilon}^p$ can be found from Equation 34 without difficulty.

The criterion used in step 6 of the iterative procedure given above, to decide whether the plastic strains have been determined with sufficient accuracy, is the size of the ratio of the increment in effective plastic strain to $\bar{\sigma}/E$. This ratio is a measure of the difference between the ordinates to the theoretical stress strain curve and the curve that is actually being used at that step in the calculation.

3. STRESS-STRAIN LAWS

The following three types of stress-strain laws are available for use in the computer program. Each of them is a three parameter law.

Type 1 - Ramberg-Osgood Law

$$\epsilon_t = \frac{\sigma}{E} + \frac{3\sigma_i}{7E} \left(\frac{\sigma}{\sigma_i} \right)^n$$

in which

E — Young's modulus

σ_i — secant yield stress (stress at which the secant modulus = 0.7E)

n — shape factor

Type 2 - Goldberg-Richard Law

$$\sigma = E \epsilon_t \left[1 + \left| \frac{E \epsilon_t}{\sigma_u} \right|^n \right]^{-1/n}$$

in which

E — Young's modulus

σ_u — maximum stress

n — shape factor

Type 3 - Bilinear Law

$$\sigma = E \epsilon_t , \quad \text{for } \sigma < \sigma_y$$

$$\sigma = \sigma_y + E_i \left(\epsilon_t - \frac{\sigma_y}{E} \right) , \quad \text{for } \sigma \geq \sigma_y$$

in which

E — Young's modulus

σ_y — yield stress

E_1 — Slope of the plastic portion of the
stress-strain curve

To reduce computing time a linearized form of the Ramberg-Osgood law is used in the program. This law is fitted by a series of straight line segments which match the actual curve at 100 points in the interval $0 \leq \epsilon_f \leq 20 \sigma_1 / E$. If values of ϵ_f outside this range are encountered the exact formula is used.

SECTION III

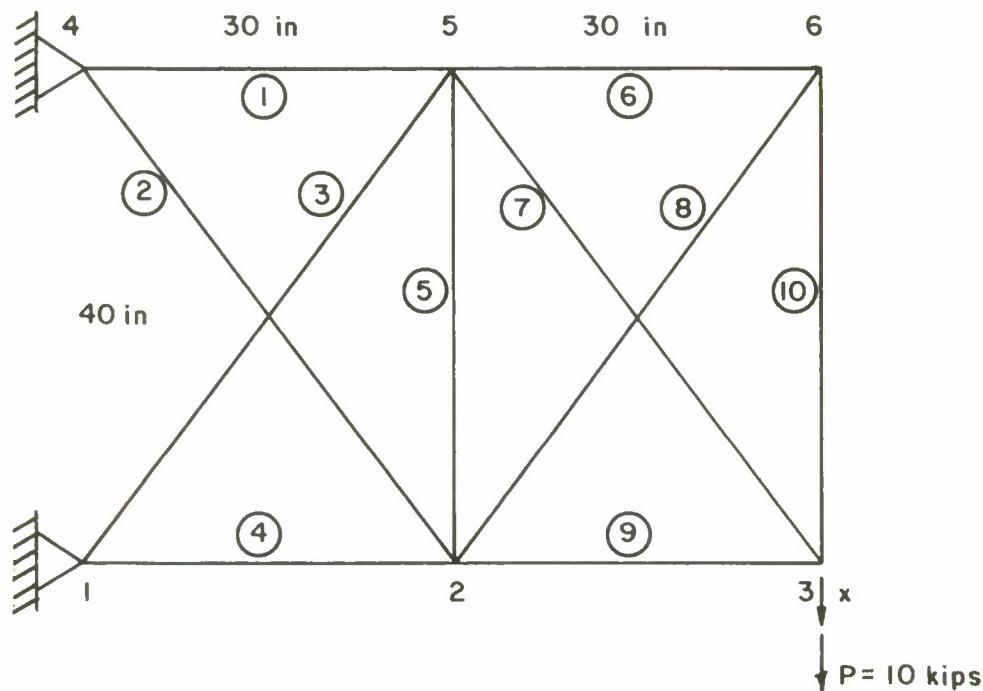
EXAMPLES

1. NONLINEAR TRUSS

To illustrate the use of the program a solution for the member stresses and the tip deflection of the cantilever truss shown in Figure 2 is obtained. A stress-strain relation of the Ramberg-Osgood type is assumed with values of $E = 10.3 \times 10^6$, $\sigma_1 = 40.5 \times 10^3$, and $n = 7$. A solution by another method is given in Reference 6. A comparison of the stresses obtained by the two methods is shown in the table in Figure 2. The tip displacement is shown as a function of the load in Figure 3. The displacements obtained by the two methods agree so closely that both solutions are represented by a single curve.

2. SHEAR LAG SPECIMEN

As a second example, solutions are obtained for the shear lag specimen tested in Reference 3 shown in Figure 4. Two solutions are obtained. In the first solution the same idealization of the structure is used as that used in Reference 3 (see Figure 5). The second solution is found using the idealization shown in Figure 6. Values of the Ramberg-Osgood constants of $E = 10.2 \times 10^6$, $\sigma_1 = 46.6 \times 10^3$, and $n = 10$ were used. These correspond to the values for the RO2 stress-strain curve of Reference 3.



ELEMENT	FORCE, in kips	
	Ref. 6	Present Analysis
1	11.26	11.18
2	6.23	6.19
3	- 6.27	- 6.31
4	-11.24	-11.11
5	- 0.51	- 0.52
6	3.35	3.33
7	6.91	6.95
8	- 5.59	- 5.55
9	- 4.15	- 4.17
10	4.47	4.44

Figure 2. Nonlinear Truss

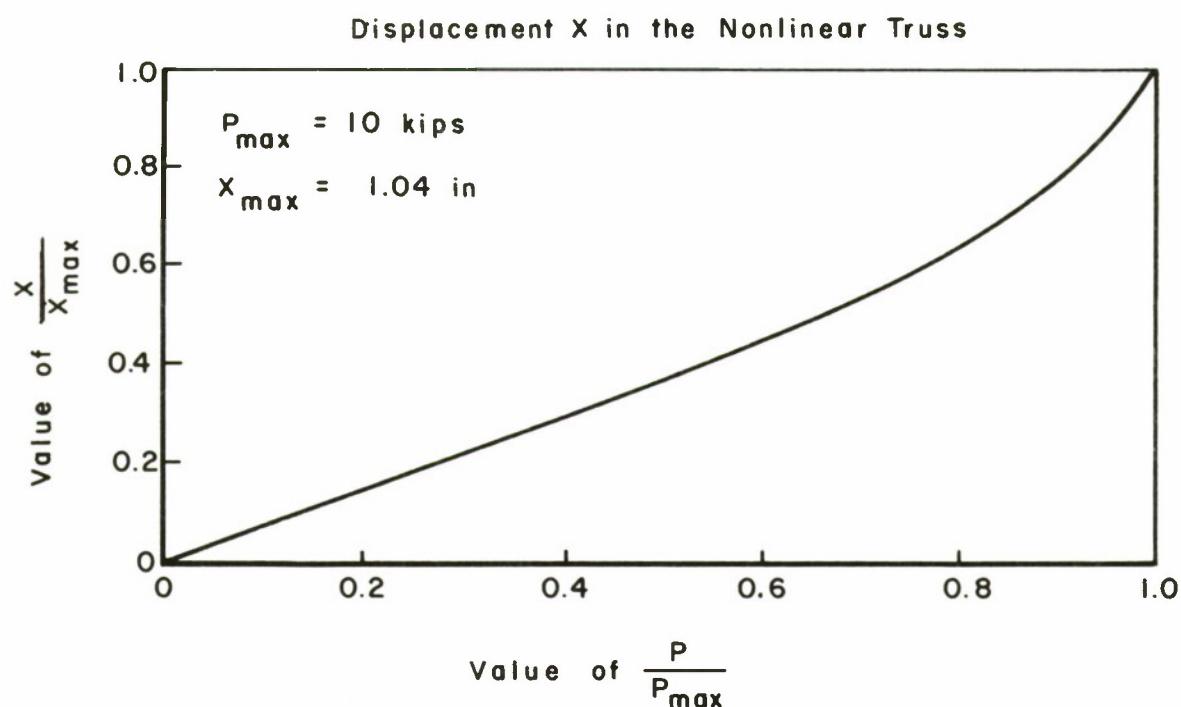


Figure 3. Load Versus Tip Displacement - Nonlinear Truss

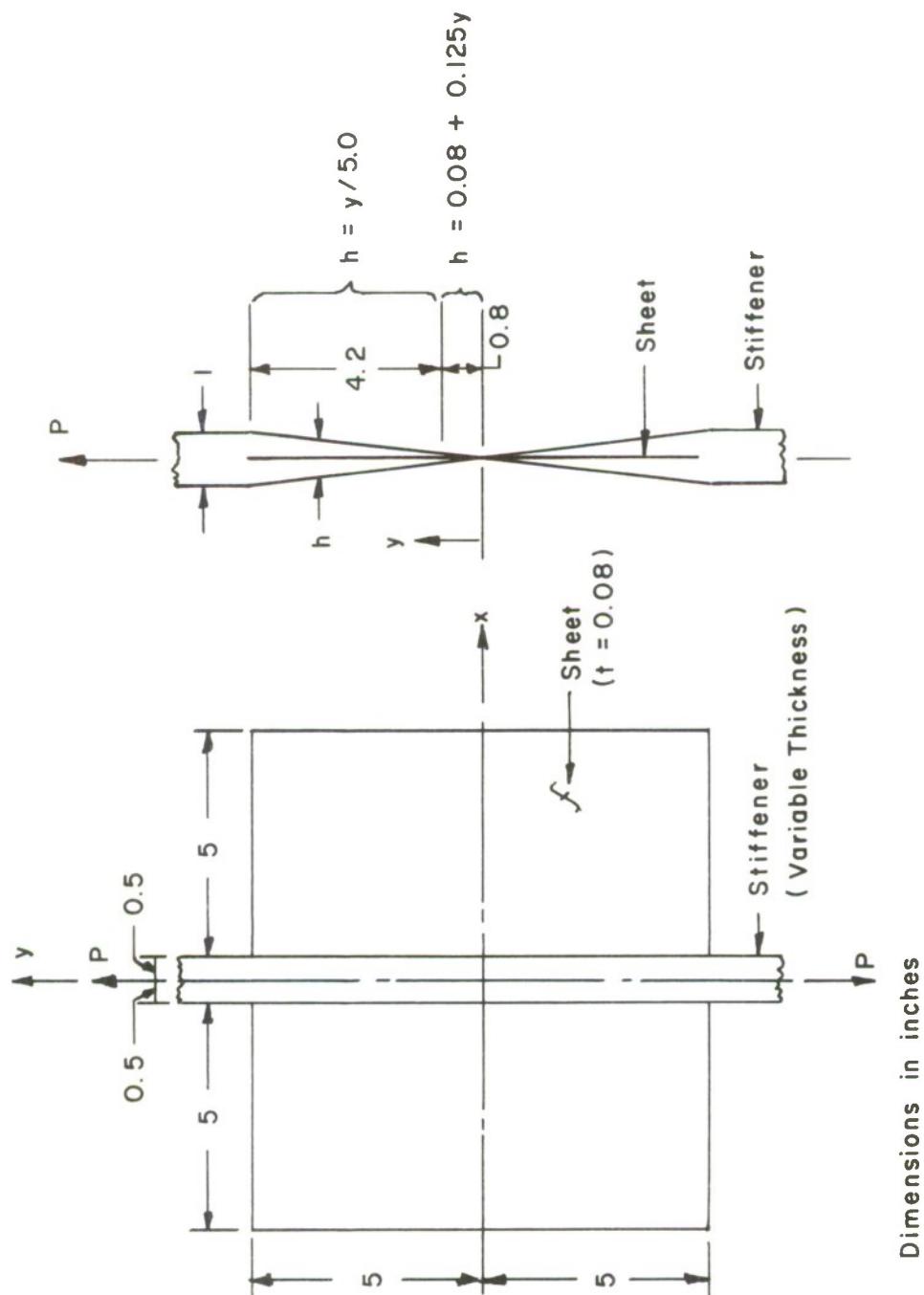


Figure 4. MIT Shear Lag Specimen

Dimensions in inches

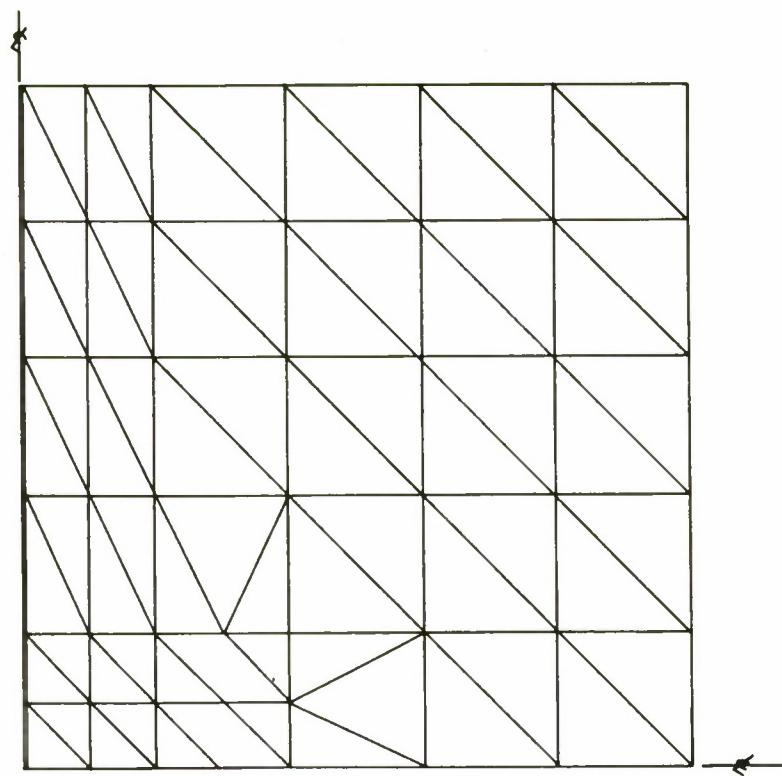


Figure 5. MIT Shear Lag Specimen - Configuration I

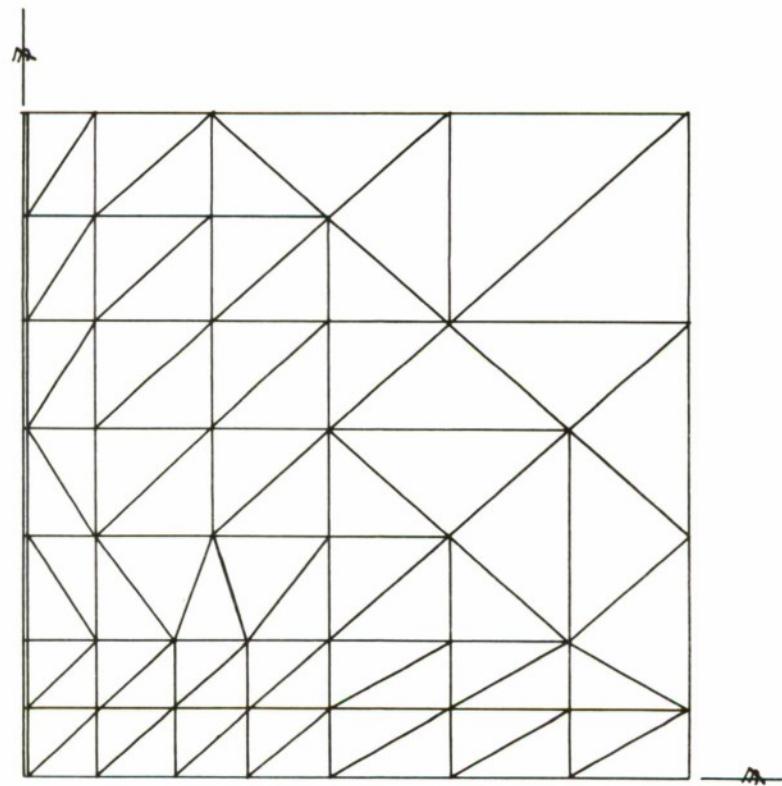


Figure 6. MIT Shear Lag Specimen - Configuration II

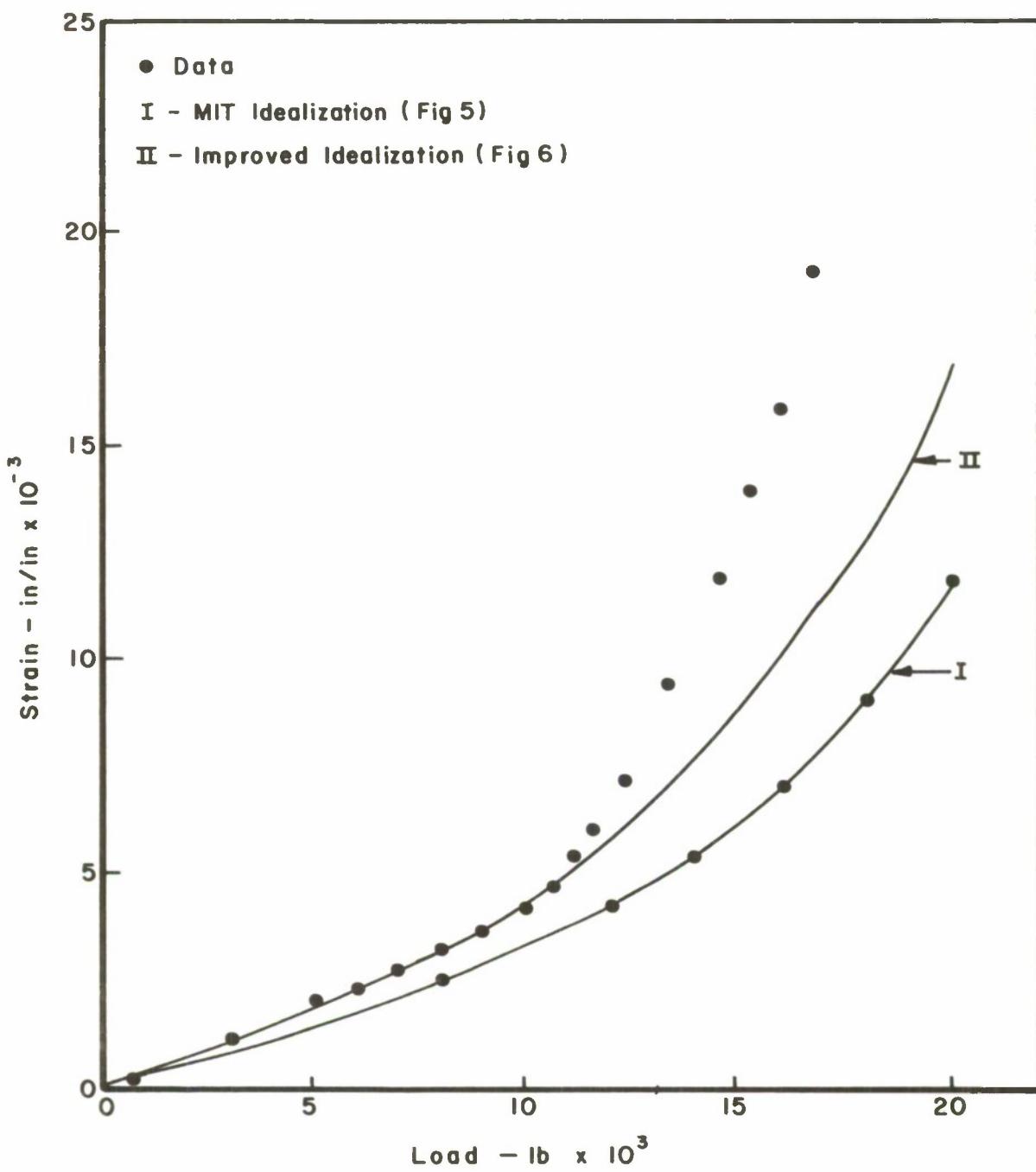


Figure 7. Comparison of Test Results of Shear Lag Specimen with Finite Element Analyses, Axial Strain at Center of Stiffener

A comparison of the results of these solutions with the test data for axial strain at the center of the stiffener is given in Figure 7. While the second solution is in better agreement with the test results than the first, the agreement at large values of the load is poor. A still more refined idealization would probably improve the solution but, as shown in Reference 5, much of the discrepancy is due to the inadequacy of the tensile stress-strain data in the range of large strains.

3. PERFORATED STRIP

Theocaris and Marketos (Reference 8) obtained results for a linear strain-hardening aluminum strip with a ratio of hole diameter to strip width of 1:2. The material properties were

$$\begin{aligned}\text{yield stress} &= 24.3 \text{ kg/mm}^2 \\ \text{plastic modulus} &= 225.0 \text{ kg/mm}^2 \\ \text{Young's modulus} &= 7000.0 \text{ kg/mm}^2\end{aligned}$$

The finite element idealization of the test specimen is shown in Figure 8; 116 nodes and 172 triangular elements were used. A comparison of measured and computed values of the maximum strain in the y direction at the edge of the hole is given in Figure 9. The agreement between theory and experiment is fairly good. The same test is used by Marcal and King (Reference 7) for comparison with the results of their analysis and about the same degree of agreement is obtained.

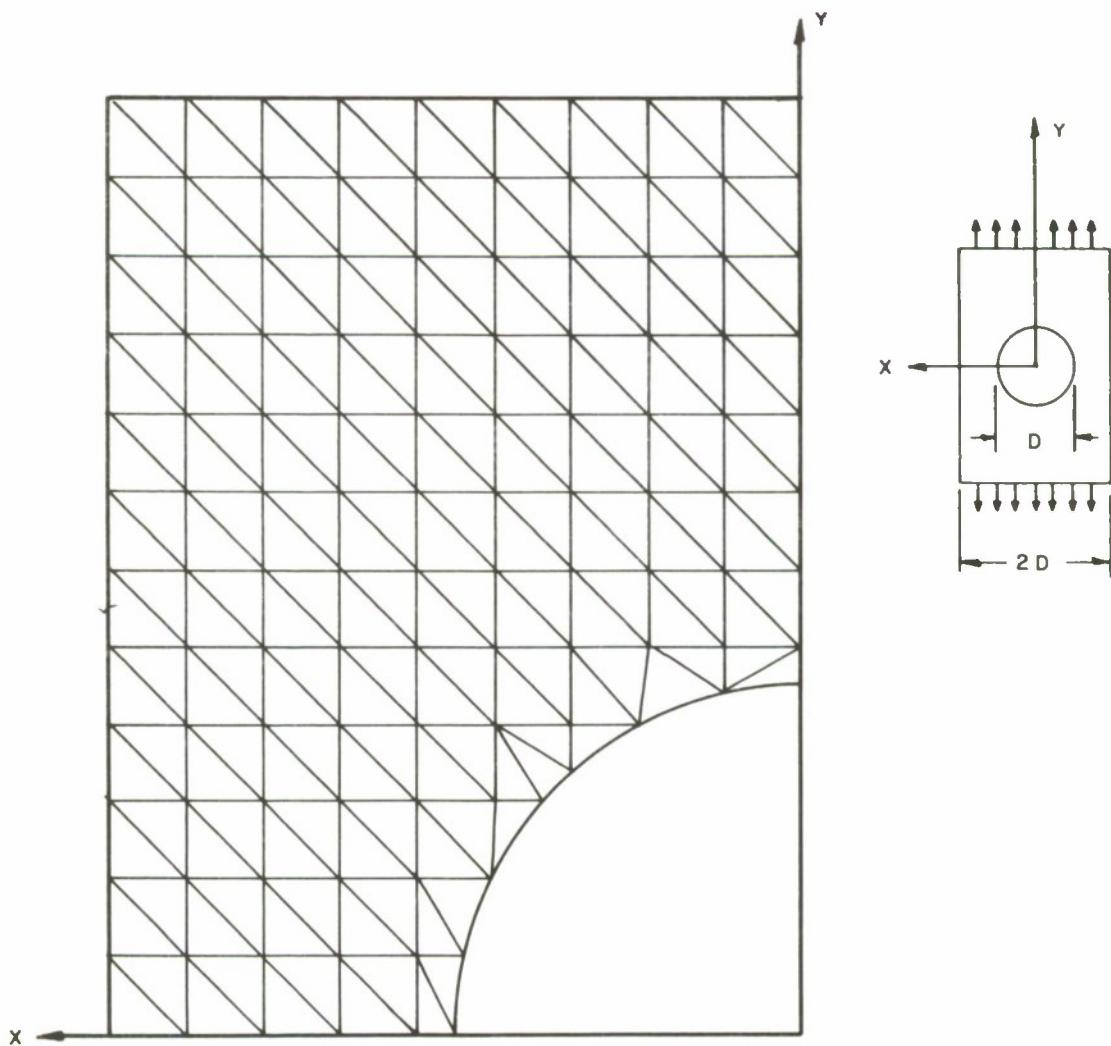


Figure 8. Perforated Strip Finite Element Idealization

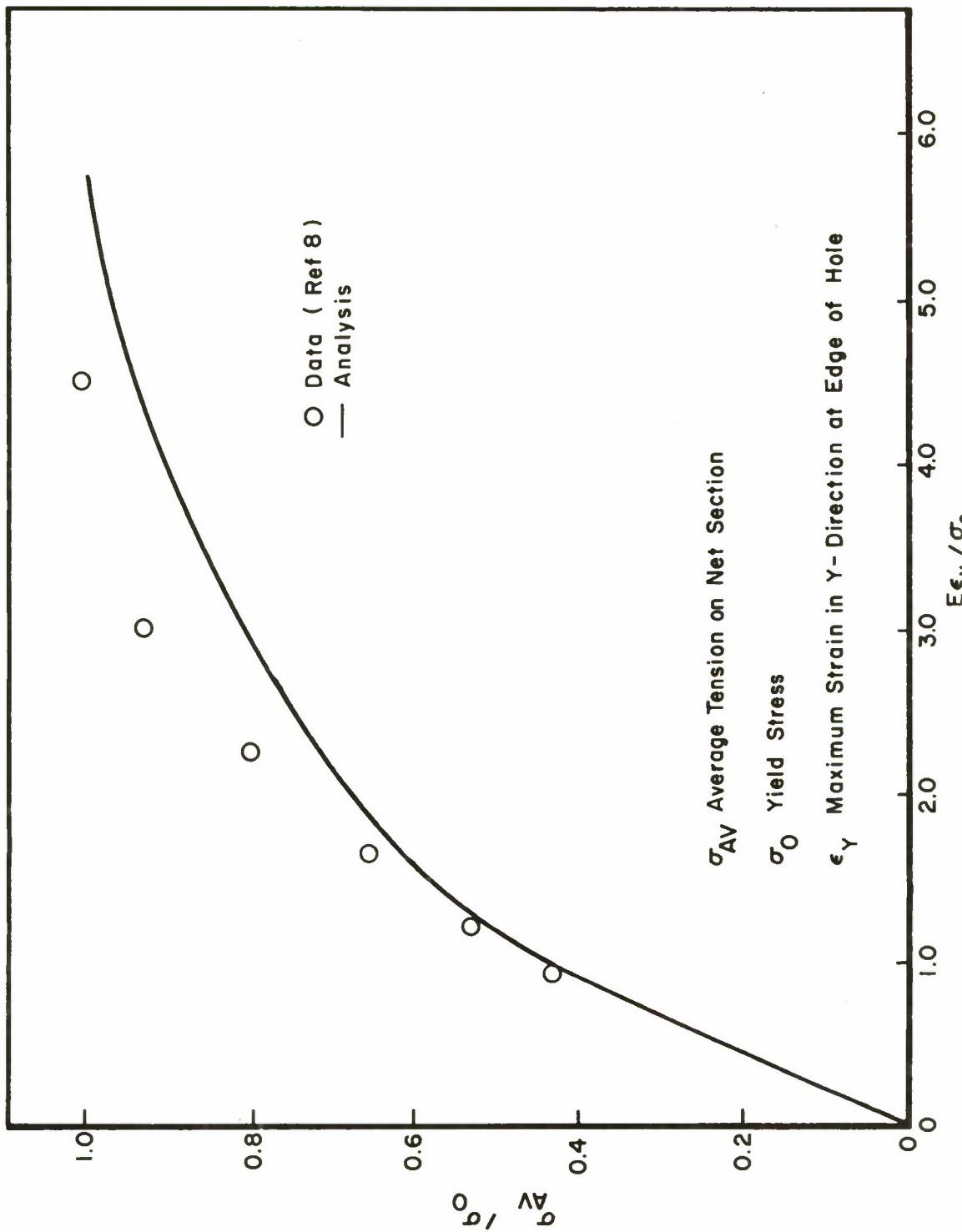


Figure 9. Maximum Values of $E\epsilon_y / \sigma_0$ for Perforated Strip

SECTION IV

DESCRIPTION OF COMPUTER PROGRAM

1. INTRODUCTION

The FORTRAN IV program for the elastic-plastic analysis of plane structures composed of bar and triangular plate elements is described in this section. The correspondence between the program variables and the stress-strain law parameters for each of the three laws available is given in Table I.

TABLE I
CORRESPONDENCE BETWEEN PROGRAM VARIABLES
AND STRESS-STRAIN LAW PARAMETERS

Stress-Strain Law	Program Variables				
	ILAW	E	EE1	EE2	PRR
Ramberg-Osgood	1	E	σ_1	n	ν
Goldberg-Richard	2	E	σ_u	n	ν
Bilinear	3	E	σ_y	E_1	ν

2. INPUT DATA AND DESCRIPTION OF OUTPUT

The geometry of the structure is determined by specifying the x and y coordinates of each node with respect to a fixed set of coordinate axes and the thickness (cross-sectional area in the case of bars) of the elements. Up to 225 nodes and 400 elements can be handled. The program uses a subroutine for the solution of simultaneous equations in band form written by Professor E. L. Wilson of the University of

California. Great economies in storage requirements and in time required for solution are achieved in this way; however, the bandwidth of the equations defined by the idealization of the structure is limited in size. To meet this limitation on bandwidth the difference between the node numbers on any element must be 9 or less. Instructions for increasing the bandwidth are given in Table II.

The displacement components in the x and y direction can be specified at any node or a node can be required to move along a line with a specified slope. Boundary conditions can be specified at up to 29 nodes.

The x and y components of load can be specified at any node. Distributed loads must be treated as concentrated at the nodes.

The number of equal increments (steps) into which the applied loads and specified displacements are to be divided is specified as input. It is also necessary to specify the number of the increment at which the solution is to start. For example, if a number of increments NDIV = 20 is specified and a value of the starting increments KSTART = 5 is used, one quarter of the load (displacement) will be applied in the first step, the rest in 15 equal increments. If it is desired to stop the solution at an intermediate step a value of KSTOP may be specified. If the unloading solution is desired the value IUNLD = 1 is used.

An error tolerance must be specified as input. After each cycle of iteration the maximum error among all the elements is compared with the specified tolerance. If the tolerance is met the next load

TABLE II
PROGRAM MODIFICATION

The bandwidth is governed by the difference between the node numbers of a given element. The largest such difference J determines the bandwidth in this program by the formula MBAND = 2 * J + 3. This number cannot be greater than 22 in the present program. This is a rather small bandwidth, but it allows 225 nodes in a 32K core machine. To change the applicable problem size of the program in terms of the basic problem size parameters the following dimensions have to be changed:

<u>To Change:</u>	<u>Change Dimensions of:</u>
1. Number of materials	EE, EE1, EE2, PRR, and TAB in common statements (presently 10)
2. Number of elements	I1, I2, I3, I4, NTYPE, Z, SEF, SET, EEP, EXPL, EYP, EXYP in common statement, JX in main program, and modify equivalence statements containing JX (presently 400)
3. Number of nodes	B, X, XCORD, Y, ICODE, FP, F in common statements, FE in main program, ICODE in subroutine DCODE, XX in subroutine ELEM (presently 225 or 2 x 225 = 450)
4. Bandwidth	B in common statements (presently 22)
5. Number of nodes with boundary conditions	BC in common statements (presently 30)

increment is applied, if not, the iteration is continued. If the tolerance on error is not met when the allowable number of iterations is reached the solution is stopped. A detailed description of the input data format is given in Table III.

TABLE III
INPUT DATA FORMAT*

Card 1	TITLE CARD (72H)		
Col	1-72 Any alphanumeric information		
Card 2	PROPERTIES CARD (1415)		
Col	1- 5 NNODE - number of nodes (maximum 225) 6-10 NELEM - number of elements (maximum 400) 11-15 ILAW - - 1 Ramberg-Osgood Law - 2 Goldberg-Richard Law - 3 Bilinear Law 16-20 IUNLD - - 1 Unloading following loading - 0 Loading only 21-25 MAT - number of materials used (maximum 10) 26-30 MAXBND - maximum bandwidth, MAXBND = 22 for this program 31-35 NBC - number of boundary conditions with prescribed displacement. The maximum number is 30 in this program.		
Card 3	MATERIAL PROPERTIES CARDS (E15. 8, 3F10. 5)		
Col	1-15 EE - modulus of elasticity 16-25 EE1 - secant yield stress, ultimate stress, yield stress 26-35 PRR - Poisson's ratio 36-45 EE2 - shape parameter, plastic modulus		
Card 4	CONTROL CARD (6I5, F10. 0)		
Col	1- 5 NDIV - number of load increments 6-10 NIT - maximum number of iterations per step 11-15 NPRINT - print output for each NPRINT incre- ment. (e. g., if NPRINT = 3, for increments 3, 6, 9 etc.) 16-20 KSTART - number of increments at which solution is to start 21-25 KSTOP - number of increments at which solution is to stop 26-30 NLOAD - number of nodes at which loads are specified 31-40 TOL - error tolerance		

TABLE III (CONTD)

Card 5	NODE	CARDS (4I5, 5F10. 0)
Col	1- 5	Node number
	6-10	IBCX = 1, if displacement in x-direction is specified
	11-15	IBCY = 1, if displacement in y-direction is specified
	16-20	IBCS = 1, if slope is specified
	21-30	XCORD - x coordinate of the node
	31-40	YCORD - y coordinate of this node
	41-50	BC1 - specified displacement in x-direction
	51-60	BC2 - specified displacement in y-direction
	61-70	BC3 - specified slope at the node
Card 6	ELEMENT	CARDS (5I5, F10. 0)
Col	1- 5	Element number
	6-10	I1 - nodes defining the element
	11-15	I2 - nodes defining the element
	16-20	I3 - nodes defining the element
	21-25	NTYPE - material type
	26-35	Z - element thickness or cross-sectional area
Card 7	LOAD	CARDS (I5, 2F10. 0)
	1- 5	Node number
	6-15	x-component of force
	16-25	y-component of force

*NOTE: Input data information in Table III is self explanatory. The use of more than one material, however, may need some clarification. The number of materials "MAT" specified in the field of card 2 defines the number of material properties cards. The sequencing of these cards in turn defines "NTYPE" in the element card, for example, if the element uses material specified in the second material properties card, integer 2 is placed in the field corresponding to "NTYPE".

The nodal forces and displacements, the maximum error and the number of the element in which it occurs are printed out at the end of each step (increment). The cartesian components, principal values, and direction of stress and strain are printed out at the user's option by specifying a value of NPRNT as input. For example, a value of NPRNT = 3 will cause the stresses and strains to be printed out for increment numbers divisible by three. The directions of the principal axes of stress are defined by

$$\phi = -\frac{1}{2} \tan^{-1} \frac{2\tau}{\sigma_x - \sigma_y}, \quad -\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

The value of ϕ in degrees is printed out. In the case of strain the principal directions are defined by

$$\phi = -\frac{1}{2} \tan^{-1} \frac{\gamma}{\epsilon_x - \epsilon_y}, \quad -\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

This value is also printed out since in general the principal axes of stress and total strain do not coincide when plastic flow has taken place.

The effective stress and the effective plastic strain are also given as output for each element.

The input data is printed out at the start of the program to aid in problem identification and checking.

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2. J. Padlog, R. D. Huff, and G. F. Holloway, Unelastic Behavior of Structures Subjected to Cyclic, Thermal, and Mechanical Stressing Conditions, WADD-TR-60-271, December 1960.
3. J. H. Percy, W. A. Loden, and D. R. Navaratna, A Study of Matrix Analysis Methods for Inelastic Structures, RDT-TDR-63-4032, October 1963.
4. J. L. Swedlow and W. H. Yang, Stiffness Analysis of Elasto-Plastic Plates, AFRPL-TR-66-5, January 1966.
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APPENDIX
COMPUTER PROGRAM LISTING

The FORTRAN IV Source Program and three sample data cases are listed. The first case is the nonlinear truss with Ramberg-Osgood representation of one material. This data is associated with Figures 2 and 3. The second case is the same truss problem slightly changed to show the introduction of more than one material. The third case is Configuration I for the MIT test specimen.

The source deck of the computer program described herein can be obtained by contacting AFFDL (FDTR/BERKE), WPAFB-Ohio, 45433. (513-25-53418).

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$IBJOB
$IBFTC VER7
C      ELASTIC PLASTIC FINITE ELEMENT PROGAM          VER70000
C      WITH THREE STRESS STRAIN LAW OPTIONS          VER70001
C      COMMON/AOD/ EE(10),EE1(10),EE2(10),PRR(10)    VER70002
C      COMMON E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,    VER70003
1      N2,NELEM,KEL,ILAW,MAT,NBC,                      VER70004
2      B(450,22),BC(30,3),TAB(101,20),IFIX(2),        VER70005
3      X(450),XCORO(225),Y(225),ICODE(225),        VER70006
4      FP(450),           VER70007
5      I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),    VER70008
6      SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),    VER70009
7      NNOOE,MBAND,                      VER70010
DIMENSION JX(400,3),FE(450)                         VER70011
EQUIVALENCE (JX,I1),(JX(401),I2),(JX( 801),I3)    VER70012
EQUIVALENCE (IFIX(1),IBCX),(IFIX(2),IBCY)          VER70013
C
C **** READ AND PRINT DATA *****
C
10 READ (5,20)                                     VER70014
  IONE=1                                         VER70015
20 FORMAT(72H BCO INFORMATION)                   VER70016
1      )                                         VER70017
      WRITE (6,30)                                VER70018
30 FORMAT(1H1)                                 VER70019
      WRITE (6,20)                                VER70020
      READ(5,40) NNOOE,NELEM,ILAW,IUNLO,MAT,MAXBND,NBC    VER70021
40 FORMAT(14I5)                                VER70022
      READ(5,50) (EE(I),EE1(I),PRR(I),EE2(I),I=1,MAT)    VER70023
50 FORMAT(E15.8,3F10.5)                          VER70024
      READ(5,60) NDIV,NIT,NPRNT,KSTART,KSTOP,NLOAD,TOL    VER70025
60 FORMAT(6I5,F10.0)                            VER70026
      IF(KSTOP.EQ.0) KSTOP=NDIV                  VER70027
      GO TO(70,90,110),ILAW                     VER70028
70 WRITE(6,80) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)    VER70029
80 FORMAT(1H014X18HRAMBERG OSGOOD LAW/          VER70030
      115X,30HMATERIAL----- I3/                VER70031
      215X30HMOULUS OF ELASTICITY----- E12.4/    VER70032
      315X30HSECANT YIELD STRESS----- E12.4/    VER70033
      415X30HSHAPE PARAMETER----- E12.4/    VER70034
      515X30HPOISSON'S RATIO----- F8.4/    VER70035
      615X30HERROR TOLERANCE----- F8.4)       VER70036
      GO TO 130                                  VER70037
90 WRITE(6,100) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)    VER70038
100 FORMAT(1H014X20HGOLOBERG RICHARD LAW/        VER70039
      115X,30HMATERIAL----- I3/                VER70040
      215X30HMOULUS OF ELASTICITY----- E12.4/    VER70041
      315X30HULTIMATE STRESS----- E12.4/    VER70042
      415X30HSHAPE PARAMETER----- E12.4/    VER70043
      515X30HPOISSON'S RATIO----- F8.4/    VER70044
      615X30HERROR TOLERANCE----- F8.4)       VER70045
      GO TO 130                                  VER70046
110 WRITE(6,120) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)    VER70047
120 FORMAT(1H014X12HBILINEAR LAW/          VER70048
      115X,30HMATERIAL----- I3/                VER70049
      215X30HMOULUS OF ELASTICITY----- E12.4/    VER70050
      315X30HYIELO STRESS----- E12.4/    VER70051
      415X30HPLASTIC MODULUS----- E12.4/    VER70052
      515X30HPOISSON'S RATIO----- F8.4/    VER70053
      615X30HERROR TOLERANCE----- F8.4)       VER70054
130 WRITE (6,140)NNOOE,NELEM,NOIV,NIT             VER70055
140 FORMAT(1H014X30HNO. OF NODES)                 NNOOE =14/15X30HNO. OF ELEMVER70060

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1ENTS      NELEM =I4/15X30HNO. OF STEPS          NDIV =I4/15X30VER70061
2HNO. OF ITERATIONS/STEP   NIT =I4)           VER70062
DO 150 I=1,NBC                         VER70063
DO 150 J=1,3                           VER70064
150 BC(I,J)=0                         VER70065
IC=1
WRITE(6,160)                           VER70066
160 FORMAT(25H0BOUNDARY CONDITION ARRAY/10HO NODAL PT15X1HX23X1HY    VER70068
     120X7HSLIDING/1H 14X4HCODE7X5HVALUE9X4HCODE7X5HVALUE9X4HCODE    VER70069
     27X5HVALUE)                           VER70070
C
C **** NODE COORDINATES AND BOUNDARY CONDITIONS ****          VER70071
C
DO 200 J=1,NNODE                      VER70072
READ(5,170) K,I8CX,I8CY,I8CS,XCORD(K),Y(K),BC1,8C2,BC3        VER70073
170 FORMAT(4I5,5F10.0)                  VER70074
IF(I8CX+IBCY+IBCS.NE.0) WRITE(6,180)K,I8CX,8C1,IBCY,BC2,IBCS,8C3  VER70075
180 FORMAT(17,3X,3(I8,1PE17.7))       VER70076
ICODE(K)=IBCS+10*IBCY+100*I8CX      VER70077
IF(8C1+BC2+BC3.EQ.0.) GO TO 200      VER70078
ICODE(K)=ICODE(K)+IC*1000            VER70079
BC(IC,1)=BC1                          VER70080
BC(IC,2)=BC2                          VER70081
BC(IC,3)=BC3                          VER70082
IC=IC+1
IF(IC.LE.NBC)GO TO 200                VER70083
WRITE(6,190)                           VER70084
190 FORMAT(54H0 MORE THAN 29 NODES HAVE NON ZERO BOUNDARY CONDITIONS)  VER70085
GO TO 620
200 CONTINUE                           VER70086
C
C **** ELEMENT PROPERTIES ****          VER70087
C
READ(5,210)(K,I1(K),I2(K),I3(K),NTYPE(K),Z(K),J=1,NELEM)        VER70088
210 FORMAT(5I5,F10.0)                  VER70089
C
C **** LOADS ****                      VER70090
C
N2=2*NNODE                           VER70091
DO 220 K=1,N2                         VER70092
220 F(K)=0                           VER70093
IF(NLOAD.EQ.0) GO TO 250              VER70094
DO 230 K=1,NLOAD                      VER70095
230 READ(5,240)J,F(2*K-1),F(2*K)      VER70096
240 FORMAT(15,2F10.0)                  VER70097
250 CONTINUE                           VER70098
WRITE(6,260)(K,XCURD(K),F(2*K-1), Y(K),F(2*K),ICODE(K),K=1,NNODE)  VER70099
260 FORMAT(10HO NODAL PT8X7HX-COORD8X7HX-FORCE8X7HY-COORD8X7HY-FORCE  VER70100
     111X4HCODE//(4X,I3,5X4F15.4,I15))  VER70101
WRITE(6,270)                           VER70102
270 FORMAT(1H0///10X,90HELEMENT      NODE 1      NODE 2      NODE 3      ELEMENT
     1T TYPE      AREA OR THICK.      MATERIAL TYPE,//)  VER70103
     DO 300 K=1,NELEM                  VER70104
     IF(I3(K).EQ.0) WRITE(6,280) K,I1(K),I2(K),I3(K),Z(K),NTYPE(K)  VER70105
     IF(I3(K).NE.0) WRITE(6,290) K,I1(K),I2(K),I3(K),Z(K),NTYPE(K)  VER70106
280 FORMAT( 6X,4I9,11X,3H8AR,E22.5,I14)  VER70107
290 FORMAT( 6X,4I9,11X,5HPLATE,E20.5,I14)  VER70108
300 CONTINUE                           VER70109
C
C INITIALIZATION                      VER70110
C
310 XDIV=NDIV                         VER70111

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      GO TO(320,340,340),ILAW          VER70123
320 CONTINUE                         VER70124
      DO 330 I=1,MAT                   VER70125
      E=EE(I)                          VER70126
      E1=EE1(I)                        VER70127
      E2=EE2(I)                        VER70128
      CC=E1/E                          VER70129
      G1=(7.0*E/3.0)**(1.0/E2)*E1**(1.0-1.0/E2)  VER70130
      CALL TABLE(I)                   VER70131
330 CONTINUE                         VER70132
340 CONTINUE                         VER70133
C
C **** DETERMINE BAND WIDTH ****    VER70134
C
      00 350 K=1,NELEM                 VER70135
      I4(K)=I3(K)                     VER70136
350 IF(I3(K).EQ.0) JX(K,3)=JX(K,1)   VER70137
      J=0                            VER70138
      00 380 N=1,NELEM                 VER70139
      DO 380 I=1,3                    VER70140
      DO 370 L=1,3                    VER70141
      KK=IABS(JX(N,I)-JX(N,L))      VER70142
      IF(KK-J)370,370,360            VER70143
      360 J=KK                         VER70144
      370 CONTINUE                     VER70145
380 CONTINUE                         VER70146
      MBAND=2*j+3                    VER70147
      IF(MBAND.GT.MAXBND) WRITE(6,390) MBAND  VER70148
      IF(MBANO.GT.MAXBND) GO TO 10        VER70149
390 FORMAT(1H010X20HBAND WIDTH TOO LARGE5X6HMBAND=14)  VER70150
      DO 400 I=1,NELEM                 VER70151
400 I3(I)=I4(I)                     VER70152
      00 410 I=1,N2                    VER70153
      00 410 J=1,MBANO                 VER70154
      410 B(I,J)=0.                   VER70155
C
C      CALCULATION OF STIFFNESS MATRIX  VER70156
C
C      CALL STIFF                      VER70157
C
C **** REDUCE MATRIX ****           VER70158
C
C      CALL SYMSOL(1)                  VER70159
C
C **** INCREMENT LOADS,AOD PLASTIC FORCES AND SOLVE FOR DISPLACEMENTS**VER70160
C
      DO 420 I=1,NELEM                 VER70161
      SET(I)=0                         VER70162
      SEF(I)=0                         VER70163
      EEP(I)=0                         VER70164
      EXPL(I)=0                         VER70165
      EYP(I)=0                         VER70166
      420 EXYP(I)=0                    VER70167
      KO=KSTART-1                     VER70168
      KU=KO                           VER70169
      DO 430 I=1,N2                    VER70170
      430 FP(I)=0                      VER70171
      GO TO 490                         VER70172
      440 WRITE (6,450)KU,(I,FE(2*I-1),FE(2*I),X(2*I-1),X(2*I),I=1,NNODE)  VER70173
      450 FORMAT(1H120X38HFORCES AND DISPLACEMENTS FOR INCREMENT,I4//10X4HNOVER70174
      1DE5X7HX-FORCE8X7HY-FORCE9X8HX-0ISPL.7X8HY-DISPL./(9XI3,2F15.3,5X2EVER70175
      215.4 ))                          VER70176

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      WRITE (6,460)XERR,KEL,IT                               VER70185
460  FURMAT(13HOMAX. ERROR =F8.5,5X14HIN ELEMENT NO.I4,5X17HNO. OF ITERVER70186
      IATIONSI4)                                         VER70187
470  IF(MOD(KU,NPRNT))490,480,490                         VER70188
480  CALL OUTPT                                         VER70189
      IF(KU.EQ.KSTOP)GO TO 620                           VER70190
      GO TO 500                                           VER70191
490  IF(KU.EQ.KSTOP) CALL OUTPT                         VER70192
      IF(KU.EQ.KSTOP)GO TO 620                           VER70193
500  KO=KU+IONE                                         VER70194
      KU=KU+1                                            VER70195
      IF(KO-NDIV)510,510,620                           VER70196
510  XK0=KO                                             VER70197
      DO 520 I=1,N2                                       VER70198
520  FE(I)=XK0/XDIV*F(I)                                VER70199
      DO 530 K=1,NELEM                                     VER70200
530  SEF(K)=SET(K)                                      VER70201
      IT=0                                               VER70202
540  DO 570 I=1,NNODE                                     VER70203
      IF(ICODE(I).EQ.0) GO TO 570                         VER70204
      CALL DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)    VER70205
      IF(IBCS.NE.1) GO TO 550                           VER70206
      ALF=BC(IC,3)                                         VER70207
      FP(IX)=FP(IX)+ALF*FP(IY)                           VER70208
      FP(IY)=0.                                           VER70209
550  DO 560 N=1,2                                         VER70210
      IF(IFIX(N).NE.1) GO TO 560                         VER70211
      IR=IX+N-1                                         VER70212
      FP(IR)=0.                                           VER70213
560  CONTINUE                                         VER70214
570  CONTINUE                                         VER70215
C
C   ***** SOLVE FOR DISPLACEMENTS *****
C
      DO 580 I=1,N2                                       VER70216
580  X(I)=FE(I)+FP(I)                                    VER70217
      CALL SYMSOL(2)                                      VER70218
C
C   CALCULATE TOTAL STRAINS,STRESSES AND PLASTIC        VER70219
C   FORCES AND STRAINS FOR EACH ELEMENT                  VER70220
C
      DO 590 I=1,N2                                       VER70221
590  FP(I)=0.                                           VER70222
      XERR=0.0                                           VER70223
      KEL=0                                              VER70224
      CALL STRAIN                                         VER70225
C
C   ***** PICK LARGEST ERROR AND DETERMINE WHEN TO REITERATE *****
C
      IT=IT+1                                           VER70226
      IF(XERR-TOL)440,440,600                           VER70227
600  IF(IT-NIT)540,610,610                           VER70228
610  KO=NDIV                                           VER70229
      IUNLD=0                                            VER70230
      GO TO 440                                         VER70231
620  IF(IUNLD.EQ.0) GO TO 10                           VER70232
      IUNLD=0                                           VER70233
      IONE=-1                                           VER70234
      KSTOP=0                                            VER70235
      GO TO 440                                         VER70236
630  END                                              VER70237
$IBFTC ELM                                         VER70238
                                                VER70239
                                                VER70240
                                                VER70241
                                                VER70242
                                                VER70243
                                                VER70244
                                                VER70245
                                                ELM 0000

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SUBROUTINE ELEM(M) ELM 0001
COMMON/ADD/ EE(10),EE1(10),EE2(10),PRR(10) ELM 0002
COMMON E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR, ELM 0003
1 N2,NELEM,KEL,ILAW,MAT,NBC, ELM 0004
2 B(450,22),BC(30,3),TAB(101,20),IFIX(2), ELM 0005
3 X(450),XCORD(225),Y(225),ICODE(225), ELM 0006
4 FP(450),F(450), ELM 0007
5 I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400), ELM 0008
6 SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400), ELM 0009
7 NNODE,MBAND ELM 0010
DIMENSION XX(225) ELM 0011
EQUIVALENCE (XCORD,XX) ELM 0012
J1=I1(M) ELM 0013
J2=I2(M) ELM 0014
J3=I3(M) ELM 0015
X21=XX(J2)-XX(J1) ELM 0016
Y21=Y(J2)-Y(J1) ELM 0017
IF(J3.EQ.0) GO TO 10 ELM 0018
Y32=Y(J3)-Y(J2) ELM 0019
Y31=Y(J3)-Y(J1) ELM 0020
X32=XX(J3)-XX(J2) ELM 0021
X31=XX(J3)-XX(J1) ELM 0022
RETURN ELM 0023
10 Y32=SQRT(X21**2+Y21**2) ELM 0024
RETURN ELM 0025
END ELM 0026
$IBFTC DCUD DCOD0000
CDCOD DCUD0001
SUBROUTINE DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC) DCOD0002
DIMENSION ICODE(225) DCOD0003
IBCS=MOD(ICODE(I),10) DCOD0004
IBCX=MOD(ICODE(I),1000)/100 DCOD0005
IBCY=MOD(ICODE(I),100)/10 DCOD0006
IC=MOD(ICODE(I),100000)/1000 DCOD0007
IX=2*I-1 DCOD0008
IY=IX+1 DCOD0009
IF(IC.EQ.0) IC=NBC DCOD0010
RETURN DCOD0011
END DCOD0012
$IBFTC PNEW PNEW0000
C2222 PLASTIC STRAIN DETERMINATION PNEW0001
SUBROUTINE PNEW1 (EEP,K,EET,E,E1,E2) PNEW0002
J=1 PNEW0003
XU=EET PNEW0004
XL=0 PNEW0005
10 EEPK=.5*(XL+XU) PNEW0006
20 Y=EEPK+E1/E*EEPK**((1.0/E2)-EET) PNEW0007
30 YP=1.0+E1/E/E2*EEPK**((1.0/E2-1.0)) PNEW0008
J=J+1 PNEW0009
IF(J>50)40,40,100 PNEW0010
40 IF(Y>50,100,60) PNEW0011
50 XL=EEPK PNEW0012
GO TO 70 PNEW0013
60 XU=EEPK PNEW0014
70 XT=EEPK-Y/YP PNEW0015
IF(XU-XT)>10,10,80 PNEW0016
80 IF(XT-XL)>10,10,90 PNEW0017
90 EEPK=XT PNEW0018
DIFF=ABS(Y/YP/EEPK) PNEW0019
IF(DIFF-.00001)>100,100,20 PNEW0020
100 RETURN PNEW0021
END PNEW0022

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$IBFTC PLSTR          PLST0000
CTABL  STRAIN-PLASTIC STRAIN TABLE          PLST0001
       SUBROUTINE TABLE(K)          PLST0002
       COMMMDN/ADD/  EE(10),EE1(10),EE2(10),PRR(10)          PLST0003
       CDMMDN   E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,          PLST0004
1      N2,NELEM,KEL,ILAW,MAT,NBC,          PLST0005
2      B(450,22),BC(30,3),TAB(101,20),IFIX(2),          PLST0006
3      X(450),XCDRD(225),Y(225),ICODE(225),          PLST0007
4      FP(450),F(450),          PLST0008
5      I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),          PLST0009
6      SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),          PLST0010
7      NNODE,MBAND          PLST0011
II2=2*K          PLST0012
II1=II2-1          PLST0013
TAB(1,II1)=0.          PLST0014
TAB(1,II2)=0.          PLST0015
DO 20 I=1,101          PLST0016
IF(I-1)20,20,10          PLST0017
10 TAB(I,II1)=FLDAT(I-1)*CC/5.          PLST0018
EET=TAB(I,II1)
CALL PNEW1(EEP,K,EET,E,G1,E2)
TAB(I,II2)=EEP
20 CDNTINUE
RETURN
END

$IBFTC STIF          STIF0000
       SUBRDUITNE STIFF          STIF0001
       CDMMDN/ADD/  EE(10),EE1(10),EE2(10),PRR(10)          STIF0002
       COMMON   E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,          STIF0003
1      N2,NELEM,KEL,ILAW,MAT,NBC,          STIF0004
2      B(450,22),BC(30,3),TAB(101,20),IFIX(2),          STIF0005
3      X(450),XCDRD(225),Y(225),ICDDE(225),          STIF0006
4      FP(450),F(450),          STIF0007
5      I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),          STIF0008
6      SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),          STIF0009
7      NNDDE,MBAND          STIF0010
DIMENSION FFP(2),INDDE(3),LNODE(6),DSK(6,6)
EQUIVALENCE (IFIX(1),IBCX),(IFIX(2),IBCY)
EQUIVALENCE (INODE(1),J1),(INODE(2),J2),(INODE(3),J3)
DO 80 M=1,NELEM
J1=I1(M)
J2=I2(M)
J3=I3(M)
CALL ELEM(M)
NTY=NTYPE(M)
E=EE(NTY)
PR=PRR(NTY)
IF(J3.EQ.0) GO TO 30
IF(NTY.LE.MAT) GO TD10
GD TO 160
C
C **** STIFFNESS MATRIX CALCULATIONS FDR TRIANGULAR ELEMENTS ****
C
10 JMAT=6          STIF0025
AE=E*Z(M)          STIF0026
A123=X21*Y31-X31*Y21          STIF0027
A123=ABS(A123)          STIF0028
ET1=AE/(2.0*A123*(1.0-PR**2))          STIF0029
ET2=AE/(4.0*A123*(1.0+PR))          STIF0030
DSK(1,1)= ET1*Y32**2          STIF0031
DSK(2,1)=-ET1*PR*Y32*X32          STIF0032
DSK(2,2)= ET1*X32**2          STIF0033
                           +ET2*X32**2
                           -ET2*Y32*X32
                           +ET2*Y32**2          STIF0034
                           -ET2*Y32**2          STIF0035
                           +ET2*Y32**2          STIF0036

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DSK(3,1)=-ET1*Y31*Y32      -ET2*X32*X31      STIF0037
DSK(3,2)= ET1*PR*Y31*X32    +ET2*Y32*X31      STIF0038
DSK(3,3)=ET1*Y31**2        +ET2*X31**2       STIF0039
DSK(4,1)= ET1*PR*Y32*X31    +ET2*Y31*X32      STIF0040
DSK(4,2)=-ET1*X31*X32      -ET2*Y31*Y32      STIF0041
DSK(4,3)=-ET1*PR*Y31*X31    -ET2*Y31*X31      STIF0042
DSK(4,4)= ET1*X31**2        +ET2*Y31**2       STIF0043
DSK(5,1)= ET1*Y21*Y32      +ET2*X32*X21      STIF0044
DSK(5,2)=-ET1*PR*Y21*X32    -ET2*Y32*X21      STIF0045
DSK(5,3)=-ET1*Y31*Y21      -ET2*X31*X21      STIF0046
DSK(5,4)= ET1*PR*Y21*X31    +ET2*X32*X21      STIF0047
DSK(5,5)= ET1*Y21**2        +ET2*X21**2       STIF0048
DSK(6,1)=-ET1*PR*Y32*X21    -ET2*Y21*X32      STIF0049
DSK(6,2)= ET1*X32*X21      +ET2*Y21*Y32      STIF0050
DSK(6,3)= ET1*PR*Y31*X21    +ET2*Y21*X31      STIF0051
DSK(6,4)=-ET1*X31*X21      -ET2*Y21*Y31      STIF0052
DSK(6,5)=-ET1*PR*Y21*X21    -ET2*Y21*X21      STIF0053
DSK(6,6)= ET1*X21**2        +ET2*Y21**2       STIF0054
DO 20 I=1,JMAT
DO 20 J=1,JMAT
20 DSK(I,J)=DSK(J,I)
GO TO 50

C **** STIFFNESS MATRIX CALCULATIONS FOR BARS ****
C
30 ET1=Z(M)*E/Y32**3
FFP(1)=X21
FFP(2)=Y21
DO 40 I=1,2
DO 40 J=1,2
DSK(I,J)=ET1*FFP(I)*FFP(J)
DSK(I+2,J)=-DSK(I,J)
DSK(I,J+2)=-DSK(I,J)
40 DSK(I+2,J+2)=DSK(I,J)
JMAT=4

C **** INCORPORATION OF ELEMENT MATRICES INTO
C           COMPLETE STIFFNESS MATRIX ****
C
50 JMAT2=JMAT/2
K=0
DO 60 I=1,JMAT2
DO 60 J=1,2
K=K+1
60 LNODE(K)=2*INODE(I)-2+J
DO 80 I=1,JMAT
KI=LNODE(I)
DO 80 J=1,JMAT
KJ=LNODE(J)
IF(KJ-KI)80,70,70
70 K=KJ-KI+1
B(KI,K)=B(KI,K)+DSK(I,J)
80 CONTINUE

C **** DISPLACEMENT BOUNDARY CONDITIONS ****
C
DO 150 I=1,NNODE
IF(ICODE(I).EQ.0) GO TO 150
CALL DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)
IF(IBCS.NE.1) GO TO 110
ALF=BC(IC,3)
B(IX,1)=B(IX,1)+ALF*(ALF*(B(IY,1)+1.)+2.*B(IX,2))

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B(IX,2)=-ALF          STIF0099
B(IY,1)=1.             STIF0100
F(IX)=ALF*F(IY)+F(IX) STIF0101
F(IY)=0.               STIF0102
KL=IX-MBAND+2         STIF0103
KU=IX+MBAND-1         STIF0104
IF(KL.LT.1) KL=1      STIF0105
IF(KU.GT.N2)KU=N2     STIF0106
DO 100 K=KL,KU        STIF0107
IF(K.EQ.IX.OR.K.EQ.IY) GO TO 100 STIF0108
IF(K.GT.IY) GO TO 90  STIF0109
L=IX-K+1              STIF0110
B(K,L)=B(K,L)+ALF*B(K,L+1) STIF0111
B(K,L+1)=0.            STIF0112
GO TO 100              STIF0113
90 L=K-IX+1             STIF0114
B(IX,L)=B(IX,L)+ALF*B(IY,L-1) STIF0115
B(IY,L-1)=0.           STIF0116
100 CONTINUE            STIF0117
110 DO 140 N=1,2        STIF0118
IF(IFIX(N).NE.1) GO TO 140 STIF0119
IR=IX+N-1              STIF0120
ML=IR-MBAND+1           STIF0121
MU=IR+MBAND-1           STIF0122
IF(ML.LT.1) ML=1        STIF0123
IF(MU.GT.N2)MU=N2       STIF0124
DO 130 M=ML,MU          STIF0125
L=IR-M+1                STIF0126
IF(L.LE.1) GO TO 120    STIF0127
F(M)=F(M)-B(M,L)*BC(IC,N) STIF0128
B(M,L)=0.                STIF0129
GO TO 130                STIF0130
120 L=M-IR+1             STIF0131
F(M)=F(M)-B(IR,L)*BC(IC,N) STIF0132
B(IR,L)=0.                STIF0133
130 CONTINUE              STIF0134
B(IR,1)=1.                STIF0135
F(IR)=BC(IC,N)           STIF0136
140 CONTINUE              STIF0137
150 CONTINUE              STIF0138
RETURN                   STIF0139
160 WRITE(6,170) M        STIF0140
170 FORMAT(1H010X32HINVALID ELEMENT CODE ELEMENT NO.I4) STIF0141
STOP                      STIF0142
END                       STIF0143
C
$IBFTC SYMSL           STIF0144
SUBROUTINE SYMSOL(KKK)   SYMS0000
COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)  SYMS0001
COMMON E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR, SYMS0002
1      N2,NELEM,KEL,ILAW,MAT,NBC, SYMS0003
2      B(450,22),BC(30,3),TAB(1C1,20),IFIX(2), SYMS0004
3      X(450),XCORD(225),Y(225),ICODE(225), SYMS0005
4      FP(450),F(450), SYMS0006
5      I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400), SYMS0007
6      SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400), SYMS0008
7      NNODE,MBAND           SYMS0009
C
NN=N2                  SYMS0010
MM=MBAND                SYMS0011
GO TO (10,60),KKK        SYMS0012
                                SYMS0013
                                SYMS0014

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C          REDUCE MATRIX                               SYMS0015
C          REDUCE MATRIX                               SYMS0016
C          REDUCE MATRIX                               SYMS0017
C          10 DO 50 N=1,NN                           SYMS0018
C             DO 40 L=2,MM                           SYMS0019
C                C=B(N,L)/B(N,1)                      SYMS0020
C                I=N+L-1                            SYMS0021
C                IF(NN-I) 40,20,20                     SYMS0022
C               20 J=0                                SYMS0023
C                  DO 30 K=L,MM                      SYMS0024
C                     J=J+1                          SYMS0025
C                     30 B(I,J)=B(I,J)-C*B(N,K)        SYMS0026
C                     40 B(N,L)=C                      SYMS0027
C                     50 CONTINUE                   SYMS0028
C                     GO TO 130                      SYMS0029
C          REDUCE VECTOR                               SYMS0030
C          REDUCE VECTOR                               SYMS0031
C          REDUCE VECTOR                               SYMS0032
C          60 DO 80 N=1,NN                           SYMS0033
C             DO 70 L=2,MM                           SYMS0034
C                I=N+L-1                            SYMS0035
C                IF(NN-I) 80,70,70                     SYMS0036
C                70 X (I)=X (I)-B(N,L)*X (N)           SYMS0037
C                80 X (N)=X (N)/B(N,1)                 SYMS0038
C          BACK SUBSTITUTION                         SYMS0039
C          BACK SUBSTITUTION                         SYMS0040
C          BACK SUBSTITUTION                         SYMS0041
C          N=NN                                    SYMS0042
C          90 N=N-1                                SYMS0043
C             IF(N) 100,130,100                      SYMS0044
C          100 DO 120 K=2,MM                         SYMS0045
C             L=N+K-1                            SYMS0046
C             IF(NN-L) 120,110,110                   SYMS0047
C             110 X (N)=X (N)-B(N,K)*X (L)           SYMS0048
C             120 CONTINUE                   SYMS0049
C             GO TO 90                      SYMS0050
C          130 RETURN                                SYMS0051
C          END                                     SYMS0052
C          END                                     SYMS0053
$IBFTC STRN
SUBROUTINE STRAIN                               SYMS0054
COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)      STRN0000
COMMON   E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,    STRN0001
1       N2,NELEM,KEL,ILAW,MAT,NBC,                  STRN0002
2       B(450,22),BC(30,3),TAB(101,20),IFIX(2),      STRN0003
3       X(450),XCORD(225),Y(225),ICODE(225),      STRN0004
4       FP(450),F(450),                          STRN0005
5       I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),  STRN0006
6       SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),  STRN0007
7       NNODE,M8AND                                STRN0008
DO 130 K=1,NELEM                                STRN0009
J1=2*I1(K)-1                                  STRN0010
J2=2*I1(K)                                    STRN0011
J3=2*I2(K)-1                                  STRN0012
J4=2*I2(K)                                    STRN0013
J5=2*I3(K)-1                                  STRN0014
J6=2*I3(K)                                    STRN0015
CALL ELEM(K)                                    STRN0016
NTY=NTYPE(K)                                    STRN0017
E=EE(NTY)                                      STRN0018
E1=EE1(NTY)                                    STRN0019

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E2=EE2( NTY)          -- THE MDM MODEL           STRN022
CC=E1/E                STRN023
PR=PRR( NTY)           STRN024
IF(ILAW.EQ.1)           MDM model             G1=(7.*E/3.)**(1./E2)*E1***(1.-1./E2)= 1,...,0   STRN025
IF(ILAW.GT.1)           G1=E1                  STRN026
EPR=E/(1.0-PR*PR)      STRN027
IF(I3(K).EQ.0)          GO TO 60                STRN028
C
C t **** TRIANGULAR ELEMENT CALCULATIONS **** sufficient condition that
C
10 A123=X21*Y31-X31*Y21                         STRN029
exists such that
SN=A123/ABS(A123)                                STRN030
EXT=(-Y32*X(J1)+Y31*X(J3)-Y21*X(J5))/A123      STRN031
EXT=(X32*X(J2)-X31*X(J4)+X21*X(J6))/A123      STRN032
EYT=(X32*X(J1)-Y32*X(J2)-X31*X(J3)+Y31*X(J4)) STRN033
+X21*X(J5)-Y21*X(J6))/A123                      STRN034
EXE=EXT-EXPL(K)                                    STRN035
EYE=EYT-EYP(K)                                    STRN036
EXYE=EXYT-EXYP(K)                                 STRN037
SX=EPR*(EXE+PR*EYE)                               STRN038
SY=EPR*(EYE+PR*EXE)                               STRN039
SXY=E/(1.0+PR)*EXYE/2.0                          STRN040
SE=SQRT(SX**2-SX*SY+SY**2+3.0*SXY**2)          STRN041
CRIT=ABS(SE)-SEF(K), and define                 STRN042
IF(CRIT)40,40,20                                   STRN043
20 EET=SE/E+EEP(K)                                STRN044
CALL STRSTN(EET,EEP,K,SETK,NTY)                   STRN045
DEEP=EEP-K-EEP(K)                                 STRN046
30 EEP(K)=EEP(K)                                  STRN047
SET(K)=SETK                                       STRN048
EXPL(K)=DEEP/SE*(SX-SY/2.0)+EXPL(K)              STRN049
EYP(K)=DEEP/SE*(SY-SX/2.0)+EYP(K)                STRN050
EXYP(K)=3.0*DEEP/SE*SXY+EXYP(K)                  STRN051
ERR=E*DEEP/SE                                     STRN052
ERR=ABS(ERR)                                     STRN053
GO TO 50                                         STRN054
40 ERR=0.0                                         STRN055
50 CONTINUE                                       STRN056
Q1=E*Z(K)/(1.0-PR**2)/2.0*SN                   STRN057
Q2=E*Z(K)/(1.0+PR)/4.0*SN                       STRN058
EXPT=EXPL(K)                                     STRN059
EYPT=EYP(K)                                     STRN060
EXYPT=EXYP(K)                                    STRN061
FP(J1)=-Q1*Y32*EXPT-Q1*Y32*PR*EYPT+Q2*X32*EXYPT+FP(J1) STRN062
FP(J2)= Q1*X32*PR*EXPT+Q1*X32*EYPT-Q2*Y32*EXYPT+FP(J2) STRN063
FP(J3)= Q1*Y31*EXPT+Q1*Y31*PR*EYPT-Q2*X31*EXYPT+FP(J3) STRN064
FP(J4)=-Q1*X31*PR*EXPT-Q1*X31*EYPT+Q2*Y31*EXYPT+FP(J4) STRN065
FP(J5)=-Q1*Y21*EXPT-Q1*Y21*PR*EYPT+Q2*X21*EXYPT+FP(J5) STRN066
FP(J6)= Q1*X21*PR*EXPT+Q1*X21*EYPT-Q2*Y21*EXYPT+FP(J6) STRN067
GO TO 110                                         STRN068
C root of necessity. Let  $\lambda \in \mathbb{R}^*, i.e. \lambda \in W_r$  for some value of  $r$ . By lemma 2, there is
C **** BAR CALCULATIONS ****                     STRN069
C suppose  $\lambda \in W_r$  where  $r$  is now considered fixed. By lemma 2, there is
60 EET=(X21*(X(J3)-X(J1))+Y21*(X(J4)-X(J2)))/Y32**2    STRN070
STRN=ABS(EET-EXPL(K))                                STRN071
SIGN=(EET-EXPL(K))/STRN                            STRN072
SE=E*STRN                                         STRN073
CRIT=SE-SEF(K) with values  $\epsilon$  and  $\eta$  we suppose that STRN074
EET=STRN+EEP(K)                                     STRN075
IF(CRIT)90,90,70                                   STRN076
70 CONTINUE                                         STRN077
CALL STRSTN(EET,EEP,K,SETK,NTY)                   STRN078

```

DEEP=EEP(K) the NDM model a necessary and sufficient condition for the system to be stable with respect to H^* is that

$$\text{EXPL}(K)=\text{EXPL}(K)+\text{SIGN}*\text{DEEP}$$

$$\text{ERR}=E*\text{DEEP}/SE$$

$$\text{ERR}=\text{ABS}(\text{ERR})$$

$$\text{GO TO } 100$$

$$90 \text{ ERR}=0.0$$

$$100 \text{ CONTINUE}$$

$$\text{EXPT}=\text{EXPL}(K)$$

$$Q1=E*Z(K)/Y32$$

$$FP(J1)=FP(J1)-Q1*X21*EXPT$$

$$FP(J2)=FP(J2)-Q1*Y21*EXPT$$

$$FP(J3)=FP(J3)+Q1*X21*EXPT$$

$$FP(J4)=FP(J4)+Q1*Y21*EXPT$$

$$110 \text{ IF}(\text{ERR}-XERR)130,130,120$$

$$120 \text{ XERR}=\text{ERR}$$

$$KEL=K$$

$$130 \text{ CONTINUE}$$

$$\text{RETURN}$$

$$\text{END}$$

\$IBFTC STRST

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SUBROUTINE STRSTN(EET,EEP,K,NTY)
COMMON/AD0/ EE(10),EE1(10),EE2(10),PRR(10)
COMMON E,CC,G1,E2,PR,EPK,X21,Y21,X31,Y31,X32,Y32,XERR,
1      N2,NELEM,KEL,ILAW,MAT,NBC,
2      8(450,22),8C(30,3),TA8(101,20),IFIX(2),
3      X(450),XCORD(225),Y(225),ICODE(225),
4      FP(450),F(450),
5      I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),
6      SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),
7      NNODE,MBAND
      GO TU(10,50,60),ILAW
10 J=5.0*EET/CC+1.0
      NT2=2*NTY
      NT1=NT2-1
      IF(J-101)20,30,30
20 EEPK=TAB(J,NT2)+(TAB(J+1,NT2)-TAB(J,NT2))*(EET-TAB(J,NT1))/(
1(TAB(J+1,NT1)-TAB(J,NT1)))
      GO TO 40
30 CALL PNEW1(EEP,K,E,G1,E2)
40 SETK=G1*EEP***(1.0/E2)
      RETURN
50 SETK=E*EET/(1.+(ABS(E*EET/G1))**E2)***(1./E2)
      EEPK=EET-SETK/E
      RETURN
60 EC=G1/E
      IF(EET-EC)70,70,80
70 EEPK=0.
      SETK=E*EET
      RETURN
80 EEPK=EET-EC
      SETK=G1+E2*EEP
      RETURN
90 EEPK=EEP
      SETK=G1+E2*EEP
      RETURN
      END
$IBFTC OPUT
C12345 OUTPUT SUBROUTINE
      SUBROUTINE OPUT
COMMON/AD0/ EE(10),EE1(10),EE2(10),PRR(10)
COMMON E,CC,G1,E2,PR,EPK,X21,Y21,X31,Y31,X32,Y32,XERR,
1      N2,NELEM,KEL,ILAW,MAT,NBC,
2      8(450,22),8C(30,3),TA8(101,20),IFIX(2),

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STRN084
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STRS0030
STRS0031
STRS0032
STRS0033
OTPU0000
OTPU0001
OTPU0002
OTPU0003
OTPU0004
OTPU0005
OTPU0006

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3      X(450),XCORD(225),Y(225),ICODE(225),          OTPU0007
4      FP(450),F(450),          OTPU0008
5      I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),  OTPU0009
6      SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),  OTPU0010
7      NNODE,MBAND          OTPU0011
L=0          OTPU0012
DO 70 K=1,NELEM          OTPU0013
C          OTPU0014
C **** TRIANGULAR ELEMENT CALCULATIONS ****          OTPU0015
C          OTPU0016
NTY=NTYPE(K)          OTPU0017
E=EE(NTY)          OTPU0018
PR= PRR(NTY)          OTPU0019
EPR=E/(1.-PR*PR)          OTPU0020
IF(I3(K).EQ.0) GO TO 70          OTPU0021
10 CALL ELEM(K)          OTPU0022
A123=X21*Y31-X31*Y21          OTPU0023
J1=2*I1(K)-1          OTPU0024
J2=2*I1(K)          OTPU0025
J3=2*I2(K)-1          OTPU0026
J4=2*I2(K)          OTPU0027
J5=2*I3(K)-1          OTPU0028
J6=2*I3(K)          OTPU0029
EXT=(-Y32*X(J1)+Y31*X(J3)-Y21*X(J5))/A123          OTPU0030
EYT=( X32*X(J2)-X31*X(J4)+X21*X(J6))/A123          OTPU0031
EXYT=(X32*X(J1)-Y32*X(J2)-X31*X(J3)+Y31*X(J4)
     +X21*X(J5)-Y21*X(J6))/A123          OTPU0032
1     OTPU0033
EXE=EXT-EXPL(K)          OTPU0034
EYE=EYT-EYP(K)          OTPU0035
EXYE=EXYT-EXYP(K)          OTPU0036
SX=EPR*(EXE+PR*EYE)          OTPU0037
SY=EPR*(EYE+PR*EXE)          OTPU0038
SXY=E/(1.0+PR)*EXYE/2.0          OTPU0039
PE2=SQRT(.5*(SX-SY))**2+SXY**2          OTPU0040
PHI=.5*ATAN2((-2.0*SXY),(SX-SY))*57.29578          OTPU0041
PH2=.5*ATAN2((-EXYT),(EXT-EYT))*57.29578          OTPU0042
PS1=.5*(SX+SY)          OTPU0043
SIGE1=PS1+PE2          OTPU0044
SIGE2=PS1-PE2          OTPU0045
PST1=.5*(EXT+EYT)          OTPU0046
PET2=SGRT(.5*(EXT-EYT))**2+EXYT**2/4.0          OTPU0047
STRE1=PST1+PET2          OTPU0048
STRE2=PST1-PET2          OTPU0049
PET2=2.0*PET2          OTPU0050
N1=I1(K)          OTPU0051
N2=I2(K)          OTPU0052
N3=I3(K)          OTPU0053
XC=XCORD(N1)+(X21+X31)/3.0          OTPU0054
YC=Y(N1)+(Y21+Y31)/3.0          OTPU0055
EPE=2.*SQRT((EXPL(K)**2+EXPL(K)*EYP(K)+EYP(K)**2+EXYP(K)**2/4.)/3.0)          OTPU0056
1)          OTPU0057
SE=SQRT(SX**2-SX*SY+SY**2+3.*SXY**2)          OTPU0058
L=L+1          OTPU0059
IF(MOD(L-1,14).GT.0,20,30)          OTPU0060
20 WRITE(6,40)          OTPU0061
30 WRITE(6,50)K,XC,YC,SX,SY,SXY,SIGE1,SIGE2,PE2,PHI,PH2,EXT,EYT, EXOTPU0062
1YT,STRE1,STRE2,PET2          OTPU0063
40 FORMAT(9H1EL. NO./5X11HCOORDINATES28X33HS T R E S S E S / S T R A OTPU0064
1I N S /8H PHI7X1HX8X1HY6X9H TAU-XX6X9H TAU-YY6X9H TAU-XOTPUS0065
2Y8X7HMAXIMUM8X7HMINIMUM6X9HMAX SHEAR )          OTPU0066
50 FORMAT(1H0I7,0PF8.3,F9.3,1P6E15.4/1H 0PF7.2,F8.2,9X1P6E15.4)          OTPU0067
WRITE(6,60) SE,EPE          OTPU0068.

```

```

60 FORMAT(1H 23H***** EFFECTIVE STRESS=E12.5,23H***** EFFECTIVE STRAIGHT
1N=E12.5)                                         OTPU0069
70 CONTINUE                                         OTPU0070
C                                                 OTPU0071
C ***** BAR CALCULATIONS *****
C                                                 OTPU0072
C                                                 OTPU0073
C                                                 OTPU0074
J=0                                                 OTPU0075
DO 120 K=1,NELEM                                 OTPU0076
NTY=NTYPE(K)                                     OTPU0077
E=EE(  NTY)                                     OTPU0078
PR=PRR(  NTY)                                     OTPU0079
J1=2*I1(K)-1                                    OTPU0080
J2=2*I1(K)                                      OTPU0081
J3=2*I2(K)-1                                    OTPU0082
J4=2*I2(K)                                      OTPU0083
IF(I3(K).NE.0) GO TO 120                         OTPU0084
80 CALL ELEM(K)                                   OTPU0085
EET=(X21*(X(J3)-X(J1))+Y21*(X(J4)-X(J2)))/Y32**2
SE=E*(EET-EXPL(K))                               OTPU0086
SEMZK=SE*Z(K)                                     OTPU0087
K1=I1(K)                                         OTPU0088
K2=I2(K)                                         OTPU0089
IF(J)100,90,100                                  OTPU0090
90 WRITE (6,130)                                 OTPU0091
J=1                                               OTPU0092
100 CONTINUE                                     OTPU0093
110 WRITE(6,140)K,K1,K2,SE,EET,SEMZK           OTPU0094
120 CONTINUE                                     OTPU0095
130 FORMAT(9HOBAR NO. 6X10HNODE NOS. 8X7HSTRESS 8X7HSTRAIN,4X,
113HMEMBER FORCES)                            OTPU0096
140 FORMAT(1H0 318,2E15.5,2X,E15.5)
      RETURN
      END
$DATA
NONLINEAR TRUSS PROC. ASCE DEC. 1965 ONE MATERIAL RAMBERG OSGOOD LAW
 6   10    1    1    1    22    30
0.10000000E+05  40.50000    0.3    7.0
 10   20    1    1    10    1    .01
  1    1    1
  2                      30.
  3                      60.
  4    1    1                  40.
  5                      30.    40.
  6                      60.    40.
  1    4    5    0    1    .25
  2    4    2    0    1    .20
  3    1    5    0    1    .20
  4    1    2    0    1    .25
  5    2    5    0    1    .20
  6    5    6    0    1    .25
  7    5    3    0    1    .20
  8    2    6    0    1    .20
  9    2    3    0    1    .25
 10   3    6    0    1    .20
  3          -10.
NONLINEAR TRUSS PROC. ASCE DEC. 1965 TWO MATERIALS RAMBERG OSGOOD LAW
 6   10    1    0    2    22    30
0.10000000E+05  65.00000    0.3    8.0
0.10000000E+05  45.00000    0.3    8.0
 10   20    1    1    10    1    .01
  1    1    1
  2                      30.

```

3 60.
 4 1 1 40.
 5 30. 40.
 6 60. 40.
 1 4 5 0 1 .25
 2 4 2 0 2 .20
 3 1 5 0 2 .20
 4 1 2 0 1 .25
 5 2 5 0 1 .20
 6 5 6 0 1 .25
 7 5 3 0 2 .20
 8 2 6 0 2 .20
 9 2 3 0 1 .25
 10 3 6 0 1 .20
 3 -10.
 MIT SHEAR LAG PROBLEM CONFIGURATION 1
 49 78 2 1 22 30
 0.10200000E+08 52000. 0.3 5.
 10 20 1 1 1 0.01
 1 1 1 0.0
 2 1 1 0.5
 3 1 1 1.0
 4 1 1 1.5
 5 1 1 2.0
 6 1 1 0.0 0.5
 7 1 1 0.5 0.5
 8 1 1 1.0 0.5
 9 1 1 1.5 0.5
 10 1 1 2.0 0.5
 11 1 1 3.0
 12 1 1 4.0
 13 1 1 5.0
 14 1 1 0.0 1.0
 15 1 1 0.5 1.0
 16 1 1 1.0 1.0
 17 1 1 1.5 1.0
 18 1 1 2.0 1.0
 19 1 1 3.0 1.0
 20 1 1 4.0 1.0
 21 1 1 5.0 1.0
 22 1 1 0.0 2.0
 23 1 1 0.5 2.0
 24 1 1 1.0 2.0
 25 1 1 2.0 2.0
 26 1 1 3.0 2.0
 27 1 1 4.0 2.0
 28 1 1 5.0 2.0
 29 1 1 0.0 3.0
 30 1 1 0.5 3.0
 31 1 1 1.0 3.0
 32 1 1 2.0 3.0
 33 1 1 3.0 3.0
 34 1 1 4.0 3.0
 35 1 1 5.0 3.0
 36 1 1 0.0 4.0
 37 1 1 0.5 4.0
 38 1 1 1.0 4.0
 39 1 1 2.0 4.0
 40 1 1 3.0 4.0
 41 1 1 4.0 4.0
 42 1 1 5.0 4.0
 43 1 1 0.0 5.0

44		0.5	5.0
45		1.0	5.0
46		2.0	5.0
47		3.0	5.0
48		4.0	5.0
49		5.0	5.0
1	1	2	6 1 .08
2	2	6	7 1 .08
3	2	3	7 1 .08
4	3	7	8 1 .08
5	3	4	8 1 .08
6	4	8	9 1 .08
7	4	5	9 1 .08
8	5	9	10 1 .08
9	5	10	11 1 .08
10	10	11	19 1 .08
11	11	12	19 1 .08
12	12	19	20 1 .08
13	12	13	20 1 .08
14	13	20	21 1 .08
15	6	7	14 1 .08
16	7	14	15 1 .08
17	7	8	15 1 .08
18	8	15	16 1 .08
19	8	9	16 1 .08
20	9	16	17 1 .08
21	9	10	17 1 .08
22	10	17	18 1 .08
23	10	18	19 1 .08
24	14	15	22 1 .08
25	15	22	23 1 .08
26	15	16	23 1 .08
27	16	23	24 1 .08
28	16	17	24 1 .08
29	17	24	25 1 .08
30	17	18	25 1 .08
31	18	19	25 1 .08
32	19	25	26 1 .08
33	19	20	26 1 .08
34	20	26	27 1 .08
35	20	21	27 1 .08
36	21	27	28 1 .08
37	22	23	29 1 .08
38	23	29	30 1 .08
39	23	24	30 1 .08
40	24	30	31 1 .08
41	24	25	31 1 .08
42	25	31	32 1 .08
43	25	26	32 1 .08
44	26	32	33 1 .08
45	26	27	33 1 .08
46	27	33	34 1 .08
47	27	28	34 1 .08
48	28	34	35 1 .08
49	29	30	36 1 .08
50	30	36	37 1 .08
51	30	31	37 1 .08
52	31	37	38 1 .08
53	31	32	38 1 .08
54	32	38	39 1 .08
55	33	39	32 1 .08
56	33	39	40 1 .08

57	33	34	40	1 .08
58	34	40	41	1 .08
59	34	35	41	1 .08
60	35	41	42	1 .08
61	36	37	43	1 .08
62	37	43	44	1 .08
63	37	38	44	1 .08
64	38	44	45	1 .08
65	38	39	45	1 .08
66	39	45	46	1 .08
67	39	40	46	1 .08
68	40	46	47	1 .08
69	40	41	47	1 .08
70	41	47	48	1 .08
71	41	42	48	1 .08
72	42	48	49	1 .08
73	1	6		10.0452
74	6	14		10.0754
75	14	22		10.15
76	22	29		10.25
77	29	36		10.35
78	36	43		10.45
43			10000.	

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13. ABSTRACT	
A computer program is presented for the small strain analysis of plane structures in the strain hardening elastic-plastic range. The finite element displacement method is used to perform the linear analyses in the iterative scheme. Bar and constant strain isotropic plane stress triangles are available for use in constructing structural idealizations. The use of ten different sets of material properties, three different material laws, and incremental proportional loading are available as options. Good correlation is shown with available test data and theoretical solutions.	
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